

# Discrete Origins of Matter

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## Abstract

We discuss models of the flavour problem and dark matter based on the discrete  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  flavour symmetry. A new class of dark-matter emerges out of these models, which is defined as the flavonic dark matter. An ultra-violet completion of these models based on the dark-technicolour paradigm is also presented.

*Keywords:* flavour problem, discrete abelian symmetries, flavonic dark matter, dark technicolour

*DOI:* 10.31526/BSM-2023.8

## 1. INTRODUCTION

The incompleteness of the standard model (SM) manifests itself in many ways. For instance, the absence of any mechanism for the origin of the fermionic mass spectrum and the mixing patterns, including that of leptons. This is known as the flavour problem. For a review of the flavour problem, see [1]. Moreover, discovery of dark matter extends the span of the incompleteness of the SM to a point where the need to go beyond the SM becomes inevitable.

In this work, we discuss some new developments to solve the problem of flavour and dark matter together through a generic  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  discrete flavour symmetry [2, 3, 4]. The ultra-violet origin of this symmetry lies in a dark-technicolour scenario where multi-fermion chiral condensates provide a solution of the flavour problem [2, 4, 5, 6]. A subset of this symmetry,  $\mathcal{Z}_N \times \mathcal{Z}_M$ , is capable of providing a dark matter candidate referred to as the ‘‘flavonic dark matter’’ together with a solution to the flavour problem [7].

We shall present our discussion along the following track: In Section 2, we present a phenomenological investigation of two proto-type  $\mathcal{Z}_N \times \mathcal{Z}_M$  flavour symmetries by deriving the bounds of the flavon field of the Froggatt-Nielsen (FN) mechanism [8] using the flavour physics data. In Section 3, we discuss a joint solution of the flavour problem and dark matter, which gives rise to the flavonic dark matter. We discuss an atypical solution to the flavour problem based on the VEV hierarchy in Section 4, which is based on the  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  flavour symmetry. A dark-technicolour paradigm providing a UV completion of the  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  flavour symmetry and the VEV hierarchy is discussed in Section 5.

## 2. $\mathcal{Z}_N \times \mathcal{Z}_M$ FLAVOUR SYMMETRY

The  $\mathcal{Z}_N \times \mathcal{Z}_M$  flavour symmetry effectively determines flavour structure of the SM, including that of neutrinos [3]. The key idea is to extend SM symmetry by introducing a complex singlet scalar flavon field  $\chi$ , which transforms under the  $SU(3)_c \times SU(2)_L \times U(1)_Y$  symmetry of the SM as

$$\chi : (1, 1, 0). \quad (1)$$

The Lagrangian that provides masses to the charged fermions of the SM reads as,

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} &= \left[ \frac{\chi (\chi^\dagger)}{\Lambda} \right]^{n_{ij}^u} y_{ij}^u \bar{\psi}_{L_i}^q \tilde{\varphi} \psi_{R_j}^u + \left[ \frac{\chi (\chi^\dagger)}{\Lambda} \right]^{n_{ij}^d} y_{ij}^d \bar{\psi}_{L_i}^q \varphi \psi_{R_j}^d \\ &+ \left[ \frac{\chi (\chi^\dagger)}{\Lambda} \right]^{n_{ij}^\ell} y_{ij}^\ell \bar{\psi}_{L_i}^\ell \varphi \psi_{R_j}^\ell + \text{H.c.} \\ &= Y_{ij}^u \bar{\psi}_{L_i}^q \tilde{\varphi} \psi_{R_j}^u + Y_{ij}^d \bar{\psi}_{L_i}^q \varphi \psi_{R_j}^d + Y_{ij}^\ell \bar{\psi}_{L_i}^\ell \varphi \psi_{R_j}^\ell + \text{H.c.} \end{aligned} \quad (2)$$

Employing the principle of minimum suppression [9], it turns out that  $\mathcal{Z}_2 \times \mathcal{Z}_5$  is the simplest realization of  $\mathcal{Z}_N \times \mathcal{Z}_M$  symmetry. We term it as the minimal  $\mathcal{Z}_N \times \mathcal{Z}_M$  model. Under this symmetry, the charges to various fields are assigned in the way, as depicted in Table 1.

It is observed that certain Yukawa couplings in the minimal model based on  $\mathcal{Z}_2 \times \mathcal{Z}_5$  flavour symmetry are not order-one, which is conventionally favoured in literature. We adopt a non-minimal form of  $\mathcal{Z}_N \times \mathcal{Z}_M$  paradigm,  $\mathcal{Z}_2 \times \mathcal{Z}_9$  flavour symmetry, where the Yukawa couplings are indeed order-one, and assign charges to the SM and flavon fields, as shown in Table 1.

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_5$	Fields	$\mathcal{Z}_2$	$\mathcal{Z}_5$	Fields	$\mathcal{Z}_2$	$\mathcal{Z}_9$	Fields	$\mathcal{Z}_2$	$\mathcal{Z}_9$
$u_R, c_R, t_R$	+	$\omega^2$	$\psi_{L_2}^q, \psi_{L_2}^\ell$	+	$\omega^4$	$u_R, t_R$	+	1	$\psi_{L_1}^q, \psi_{L_1}^\ell$	+	$\omega$
$d_R, s_R, b_R$	-	$\omega$	$\psi_{L_3}^q, \psi_{L_3}^\ell$	+	$\omega^2$	$c_R$	+	$\omega^4$	$\psi_{L_2}^q, \psi_{L_2}^\ell$	+	$\omega^8$
$e_R, \mu_R, \tau_R$	-	$\omega$	$\chi$	-	$\omega$	$d_R, s_R, b_R$	-	$\omega^3$	$\psi_{L_3}^q, \varphi$	+	1
$\nu_{e_R}, \nu_{\mu_R}, \nu_{\tau_R}$	-	$\omega^3$	$\varphi$	+	1	$e_R, \mu_R, \tau_R$	-	$\omega^3$	$\psi_{L_3}^\ell$	+	$\omega^6$
$\psi_{L_1}^q, \psi_{L_1}^\ell$	+	$\omega$				$\nu_{e_R}, \nu_{\mu_R}, \nu_{\tau_R}$	-	$\omega^7$	$\chi$	-	$\omega$

TABLE 1: Assigned charges to SM and flavon fields under  $\mathcal{Z}_2$ ,  $\mathcal{Z}_5$ , and  $\mathcal{Z}_9$  symmetries, where  $\omega$  is the fifth or ninth root of unity.

### 2.1. Masses and Mixing Patterns

The minimal model based on  $\mathcal{Z}_2 \times \mathcal{Z}_5$  flavour symmetry allows following mass matrices for up and down-type quarks, and charged leptons,

$$\mathcal{M}_u = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^u \epsilon^4 & y_{12}^u \epsilon^4 & y_{13}^u \epsilon^4 \\ y_{21}^u \epsilon^2 & y_{22}^u \epsilon^2 & y_{23}^u \epsilon^2 \\ y_{31}^u & y_{32}^u & y_{33}^u \end{pmatrix}, \quad \mathcal{M}_d = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^d \epsilon^5 & y_{12}^d \epsilon^5 & y_{13}^d \epsilon^5 \\ y_{21}^d \epsilon^3 & y_{22}^d \epsilon^3 & y_{23}^d \epsilon^3 \\ y_{31}^d \epsilon & y_{32}^d \epsilon & y_{33}^d \epsilon \end{pmatrix}, \quad \mathcal{M}_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^\ell \epsilon^5 & y_{12}^\ell \epsilon^5 & y_{13}^\ell \epsilon^5 \\ y_{21}^\ell \epsilon^3 & y_{22}^\ell \epsilon^3 & y_{23}^\ell \epsilon^3 \\ y_{31}^\ell \epsilon & y_{32}^\ell \epsilon & y_{33}^\ell \epsilon \end{pmatrix}. \quad (3)$$

And for the non-minimal  $\mathcal{Z}_2 \times \mathcal{Z}_9$  flavour symmetry, the corresponding mass matrices are

$$M_u = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^u \epsilon^8 & y_{12}^u \epsilon^6 & y_{13}^u \epsilon^8 \\ y_{21}^u \epsilon^8 & y_{22}^u \epsilon^4 & y_{23}^u \epsilon^8 \\ y_{31}^u & y_{32}^u \epsilon^4 & y_{33}^u \end{pmatrix}, \quad M_d = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^d \epsilon^7 & y_{12}^d \epsilon^7 & y_{13}^d \epsilon^7 \\ y_{21}^d \epsilon^5 & y_{22}^d \epsilon^5 & y_{23}^d \epsilon^5 \\ y_{31}^d \epsilon^3 & y_{32}^d \epsilon^3 & y_{33}^d \epsilon^3 \end{pmatrix}, \quad M_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^\ell \epsilon^7 & y_{12}^\ell \epsilon^7 & y_{13}^\ell \epsilon^7 \\ y_{21}^\ell \epsilon^5 & y_{22}^\ell \epsilon^5 & y_{23}^\ell \epsilon^5 \\ y_{31}^\ell \epsilon^3 & y_{32}^\ell \epsilon^3 & y_{33}^\ell \epsilon^3 \end{pmatrix}. \quad (4)$$

The masses of the quarks and charged leptons are obtained in terms of the expansion parameter  $\epsilon$ . Their expressions at the leading-order take up the form as shown in Table 2 [9, 10].

Masses	$\mathcal{Z}_2 \times \mathcal{Z}_5$	$\mathcal{Z}_2 \times \mathcal{Z}_9$
$\{m_t, m_c, m_u\}$	$\simeq \{ y_{33}^u ,  y_{22}^u  \epsilon^2,  y_{11}^u  \epsilon^4\} v / \sqrt{2}$	$\simeq \{ y_{33}^u ,  y_{22}^u  \epsilon^4,  y_{11}^u  \epsilon^8\} v / \sqrt{2}$
$\{m_b, m_s, m_d\}$	$\simeq \{ y_{33}^d  \epsilon,  y_{22}^d  \epsilon^3,  y_{11}^d  \epsilon^5\} v / \sqrt{2}$	$\simeq \{ y_{33}^d  \epsilon^3,  y_{22}^d  \epsilon^5,  y_{11}^d  \epsilon^7\} v / \sqrt{2}$
$\{m_\tau, m_\mu, m_e\}$	$\simeq \{ y_{33}^\ell  \epsilon,  y_{22}^\ell  \epsilon^3,  y_{11}^\ell  \epsilon^5\} v / \sqrt{2}$	$\simeq \{ y_{33}^\ell  \epsilon^3,  y_{22}^\ell  \epsilon^5,  y_{11}^\ell  \epsilon^7\} v / \sqrt{2}$

TABLE 2: Leading order mass patterns of quarks and leptons in terms of parameter  $\epsilon$  for minimal and non-minimal models.

The quark mixing angles up to the leading order are approximately as given in Table 3 [10].

Quark mixing angles	$\mathcal{Z}_2 \times \mathcal{Z}_5$	$\mathcal{Z}_2 \times \mathcal{Z}_9$
$\sin \theta_{12} \simeq  V_{us} $	$\simeq \left  \frac{y_{12}^d}{y_{22}^d} - \frac{y_{12}^u}{y_{22}^u} \right  \epsilon^2$	$\simeq \left  \frac{y_{12}^d}{y_{22}^d} - \frac{y_{12}^u}{y_{22}^u} \right  \epsilon^2$
$\sin \theta_{23} \simeq  V_{cb} $	$\simeq \left  \frac{y_{23}^d}{y_{33}^d} - \frac{y_{23}^u}{y_{33}^u} \right  \epsilon^2$	$\simeq \left  \frac{y_{23}^d}{y_{33}^d} \right  \epsilon^2$
$\sin \theta_{13} \simeq  V_{ub} $	$\simeq \left  \frac{y_{13}^d}{y_{33}^d} - \frac{y_{12}^d y_{23}^d}{y_{22}^d y_{33}^d} - \frac{y_{13}^u}{y_{33}^u} \right  \epsilon^4$	$\simeq \left  \frac{y_{13}^d}{y_{33}^d} - \frac{y_{12}^d y_{23}^d}{y_{22}^d y_{33}^d} \right  \epsilon^4$

TABLE 3: Leading order expressions for quark mixing angles in terms of the parameter  $\epsilon$  for minimal and non-minimal models.

For our numerical analysis, the values of the expansion parameter  $\epsilon$  turn out to be 0.1 and 0.23 for the minimal and non-minimal models, respectively [9].

### 2.2. The Scalar Potential

The scalar potential of the model can be written as

$$-V = -\mu^2 \varphi^\dagger \varphi + \lambda \left( \varphi^\dagger \varphi \right)^2 - \mu_\chi^2 \chi^* \chi + \lambda_\chi (\chi^* \chi)^2 + \left( \rho \chi^2 + \text{H.c.} \right) + \lambda_{\varphi\chi} (\chi^* \chi) \left( \varphi^\dagger \varphi \right). \quad (5)$$

Under the assumption that there is no Higgs-flavon mixing,  $\lambda_{\varphi\chi} = 0$ , flavon field can be parametrized by excitations around its VEV in the following way:

$$\chi(x) = \frac{f + s(x) + ia(x)}{\sqrt{2}}, \quad (6)$$

where  $f$  represents the VEV of the flavon field, while  $s$  and  $a$  denote its scalar and pseudoscalar components, respectively.

The minimization conditions for the scalar potential yield the following expressions for the masses of the scalar and pseudoscalar components of the flavon,

$$\begin{aligned} m_s &= \sqrt{\mu_\chi - 2\rho} = \sqrt{\lambda_\chi f}, \\ m_a &= \sqrt{-2\rho}. \end{aligned} \quad (7)$$

It is evident from equation (7) that the mass of the pseudoscalar is contingent on the soft-symmetry breaking parameter  $\rho$ , establishing it as a free parameter of the model, alongside the VEV  $f$ .

### 2.3. Bounds on the Flavour Scale of the $\mathcal{Z}_2 \times \mathcal{Z}_M$ Flavour Symmetry

Now we discuss the flavour bounds on the parameter space of the  $\mathcal{Z}_2 \times \mathcal{Z}_M$  flavour symmetries [9]. We show the summary of the most stringent bounds on the parameter space of both the minimal and non-minimal  $\mathcal{Z}_2 \times \mathcal{Z}_M$  models, where  $M = 5, 9$ , in Figure 1.

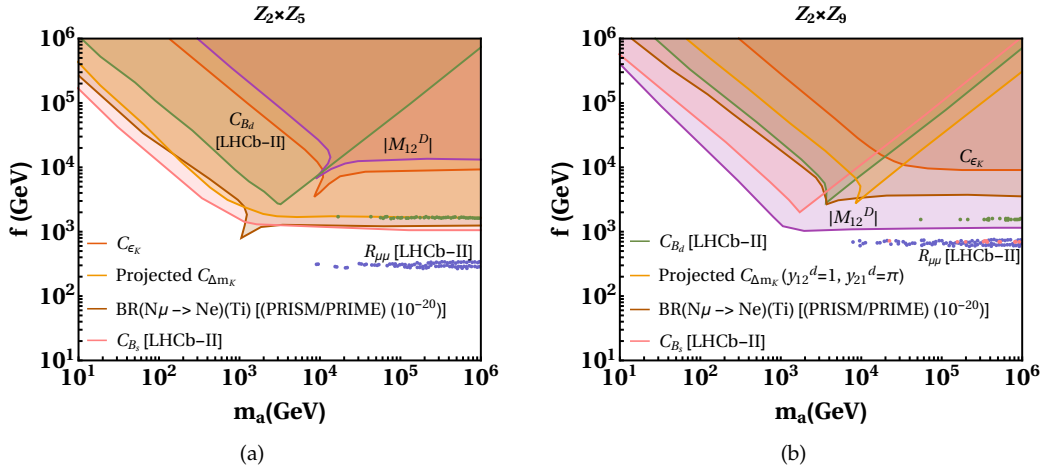


FIGURE 1: Summarized representation of constrained  $m_a - f$  parameter space for both the minimal  $\mathcal{Z}_2 \times \mathcal{Z}_5$  and non-minimal  $\mathcal{Z}_2 \times \mathcal{Z}_9$  models, depicted in the left and right panels, respectively.

Currently, the quark flavour observables from  $K^0 - \bar{K}^0$  mixing and  $D^0 - \bar{D}^0$  mixing impose stringent constraints on the parameter space for both the minimal and non-minimal models. The observable  $R_{\mu\mu}$ , which is ratio of  $BR(B_d \rightarrow \mu^+\mu^-)$  and  $BR(B_s \rightarrow \mu^+\mu^-)$  turns out to be crucial in the phase-II of the LHCb as it eliminates a large parameter space, as shown in Figure 1.

## 3. FLAVONIC DARK MATTER

The  $\mathcal{Z}_N \times \mathcal{Z}_M$  flavour symmetry framework unveils a remarkable scenario by offering a novel configuration to address both the flavour problem and the existence of dark matter within a unified and comprehensive framework. This is achieved by adding the following term to the scalar potential [7],

$$V = -\lambda \frac{\chi^{\tilde{N}}}{\Lambda^{\tilde{N}-4}} + \text{H.c.}, \quad (8)$$

where  $\lambda = |\lambda|e^{i\alpha}$ , and  $\tilde{N}$  denotes the least common multiple of  $N$  and  $M$  in the  $\mathcal{Z}_N \times \mathcal{Z}_M$  framework. For a value of  $\tilde{N}$  being larger than a particular threshold, the axial flavon field becomes light enough to be a DM candidate. We refer to it as the flavonic dark matter (FDM) [7].

The flavon attains VEV as  $\langle \chi \rangle = f/\sqrt{2}$ , leading to  $\mathcal{Z}_N$  symmetry breaking. The axial flavon mass is expressed as [7],

$$m_a^2 = \frac{1}{8} |\lambda| \tilde{N}^2 e^{\tilde{N}-4} f^2. \quad (9)$$

To achieve the cold dark matter density correctly, the mass of the flavonic dark matter is determined as,

$$m_a = 3.4 \times 10^{-3} \text{ eV} \left( \frac{10^{12} \text{ GeV}}{\varphi_0} \right)^4, \quad (10)$$

where  $\varphi_0$  represents the initial misalignment of the axial flavon field from its true vacuum during the inflationary phase. For details, see [7]. Taking  $\varphi_0 = a_0 f / \tilde{N}$ , the required axial flavon mass  $m_a$ , and VEV  $f$  turn out to be,

$$m_a = 0.88 \times 10^{16} \left( \epsilon^{\tilde{N}-4} \tilde{N}^4 \frac{|\lambda|}{a_0^2} \right)^{2/5} \text{ eV}, \quad (11)$$

$$f = 2.5 \times 10^7 \left( \frac{\tilde{N}^6}{a_0^8 |\lambda| \epsilon^{\tilde{N}-4}} \right)^{1/10} \text{ GeV}. \quad (12)$$

We utilize a framework based on  $\mathcal{Z}_8 \times \mathcal{Z}_{22}$  flavour symmetry that furnishes a set-up to impart dark-matter candidate with simultaneously addressing the flavour problem. Unlike the two prototypes discussed in Section 2, the  $\mathcal{Z}_8 \times \mathcal{Z}_{22}$  flavour symmetry prohibits the generation of mass of the top quark through tree-level SM Yukawa operator rather, they are generated by the non-renormalizable dimension-5 operator.

Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$	Fields	$\mathcal{Z}_8$	$\mathcal{Z}_{22}$
$u_R$	$\omega^2$	$\omega^2$	$c_R$	$\omega^5$	$\omega^5$	$t_R$	$\omega^6$	$\omega^6$	$d_R$	$\omega^3$	$\omega^3$	$s_R$	$\omega^4$	$\omega^4$
$b_R$	$\omega^4$	$\omega^4$	$\psi_{L,1}^q$	$\omega^2$	$\omega^{10}$	$\psi_{L,2}^q$	$\omega$	$\omega^9$	$\psi_{L,3}^q$	$\omega^7$	$\omega^7$	$\psi_{L,1}^\ell$	$\omega^3$	$\omega^3$
$\psi_{L,2}^\ell$	$\omega^2$	$\omega^2$	$\psi_{L,3}^\ell$	$\omega^2$	$\omega^2$	$e_R$	$\omega^4$	$\omega^{12}$	$\mu_R$	$\omega^7$	$\omega^7$	$\tau_R$	$\omega^7$	$\omega^{21}$
$\nu_{eR}$	$\omega^2$	1	$\nu_{\mu R}$	$\omega^5$	$\omega^3$	$\nu_{\tau R}$	$\omega^6$	$\omega^4$	$\chi$	$\omega$	$\omega$	$H$	1	1

TABLE 4: Assignment of charges to the SM and the flavon fields under the  $\mathcal{Z}_8$  and  $\mathcal{Z}_{22}$  symmetries, where  $\omega$  is the 8th or 22nd root of unity.

Following the charge assignments to the SM and flavon fields under these symmetries, as given in Table 4, the mass matrices for the charged fermions can be written as,

$$M_u = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^u \epsilon^8 & y_{12}^u \epsilon^5 & y_{13}^u \epsilon^4 \\ y_{21}^u \epsilon^7 & y_{22}^u \epsilon^4 & y_{23}^u \epsilon^3 \\ y_{31}^u \epsilon^5 & y_{32}^u \epsilon^2 & y_{33}^u \epsilon \end{pmatrix}, \quad M_d = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^d \epsilon^7 & y_{12}^d \epsilon^6 & y_{13}^d \epsilon^6 \\ y_{21}^d \epsilon^6 & y_{22}^d \epsilon^5 & y_{23}^d \epsilon^5 \\ y_{31}^d \epsilon^4 & y_{32}^d \epsilon^3 & y_{33}^d \epsilon^3 \end{pmatrix}, \quad M_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^\ell \epsilon^9 & y_{12}^\ell \epsilon^4 & y_{13}^\ell \epsilon^4 \\ y_{21}^\ell \epsilon^{10} & y_{22}^\ell \epsilon^5 & y_{23}^\ell \epsilon^3 \\ y_{31}^\ell \epsilon^8 & y_{32}^\ell \epsilon^5 & y_{33}^\ell \epsilon^3 \end{pmatrix}. \quad (13)$$

The approximate expressions for masses of the quarks and leptons at the leading-order are [7],

$$\begin{aligned} \{m_t, m_c, m_u\} &\simeq \left\{ |y_{33}^u| \epsilon, |y_{22}^u| \epsilon^4, |y_{11}^u| \epsilon^8 \right\} v / \sqrt{2}, \\ \{m_b, m_s, m_d\} &\simeq \left\{ |y_{33}^d| \epsilon^3, |y_{22}^d| \epsilon^5, |y_{11}^d| \epsilon^7 \right\} v / \sqrt{2}, \\ \{m_\tau, m_\mu, m_e\} &\simeq \left\{ |y_{33}^\ell| \epsilon^3, |y_{22}^\ell| \epsilon^5, |y_{11}^\ell| \epsilon^9 \right\} v / \sqrt{2}. \end{aligned} \quad (14)$$

The quark mixing angles are determined as [7],

$$\begin{aligned} \sin \theta_{12} &\simeq |V_{us}| \simeq \left| \frac{y_{12}^d}{y_{22}^d} - \frac{y_{12}^u}{y_{22}^u} \right| \epsilon, \\ \sin \theta_{23} &\simeq |V_{cb}| \simeq \left| \frac{y_{23}^d}{y_{33}^d} - \frac{y_{23}^u}{y_{33}^u} \right| \epsilon^2, \\ \sin \theta_{13} &\simeq |V_{ub}| \simeq \left| \frac{y_{13}^d}{y_{33}^d} - \frac{y_{12}^u y_{23}^d}{y_{22}^u y_{33}^d} - \frac{y_{13}^u}{y_{33}^u} \right| \epsilon^3. \end{aligned} \quad (15)$$

For  $\mathcal{Z}_8 \times \mathcal{Z}_{22}$  flavour symmetry, the value of  $\tilde{N}$  is determined to be 88, which using in equations (11) and (12) result in,

$$f \approx 1.0 \times 10^{14} \text{ GeV}, \quad m_a \approx 1.9 \times 10^{-3} \text{ eV}, \quad (16)$$

assuming  $\epsilon = 0.225$  with  $|\lambda| = 1$  and  $a_0 = 1$ . To ensure the prolonged stability of flavonic dark matter, its decay into electrons and positrons pair must be prohibited, implying  $m_a < 2m_e$ . This condition sets a rigorous constraint on the parameter  $\tilde{N}$  and VEV  $f$  for the existence of flavonic dark matter, specifically requiring that [7],

$$\tilde{N} > 53, \quad f > 4 \times 10^{11} \text{ GeV}. \quad (17)$$

The decays of FDM to neutrino is highly suppressed due to coupling of the neutrinos to DM being of the order of  $\epsilon^{20}$ . The axial coupling of photon to DM turns out to be  $g_{\varphi\gamma\gamma} = \frac{\alpha}{2\pi v_F} \frac{5}{3}$ , which results in the corresponding decay width  $\Gamma(a \rightarrow \gamma\gamma) = 1.69 \times$

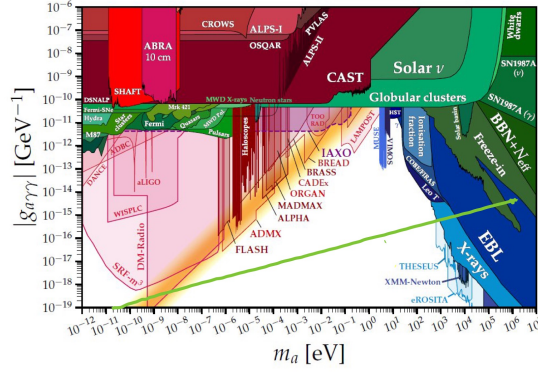


FIGURE 2: Predicted mass ranges for detection of flavonic dark matter (depicted by the thick green line) and axion-like particles [14].

$10^{-68}$  GeV [7]. These results suggest that the lifetimes of these processes exceed the age of the universe, indicating the stability of flavonic dark matter.

The predicted photon coupling versus the flavonic dark matter (FDM) mass is presented in Figure 2. It reveals that FDM masses exceeding approximately 1 keV, corresponding to  $\tilde{N} < 67$  and  $v_F < 4 \times 10^{12}$  GeV, are excluded. This exclusion aligns with recent constraints from INTEGRAL/SPI data [11]. Notably, these predictions also coincide with that of the GUT-scale QCD axion mass at around  $10^{-9}$  eV, leading to a prospect for investigations in future [12]. The mass range above the keV level can be explored further through the upcoming THESEUS experiment [13].

#### 4. THE FRAMEWORK OF THE VEV HIERARCHY

In this section, we turn up to an approach that stands in stark contrast to the FN mechanism relying on  $\mathcal{Z}_N \times \mathcal{Z}_M$  flavour symmetry but shares the use of discrete symmetries. The Hierarchical VEVs model (HVM) firstly proposed in [4] redefines the flavour problem by incorporating three Abelian discrete symmetries  $\mathcal{Z}_2$ ,  $\mathcal{Z}'_2$ ,  $\mathcal{Z}''_2$  and six gauge singlet scalar fields  $\chi_i$  ( $i = 1 - 6$ ), whose VEVs account for both the fermionic mass spectrum and flavour mixing. The masses of the SM fermions are produced by the following dimension-5 operators,

$$\mathcal{L} = \frac{1}{\Lambda} \left[ y_{ij}^u \bar{\psi}_{L_i}^q \tilde{\phi} \psi_{R_j}^u \chi_i + y_{ij}^d \bar{\psi}_{L_i}^q \phi \psi_{R_j}^d \chi_{i+3} + y_{ij}^l \bar{\psi}_{L_i}^l \phi \psi_{R_j}^l \chi_{i+3} \right] + \text{H.c.} \quad (18)$$

The hierarchy in the masses and mixing patterns is explained by VEVs hierarchy in a way that  $\langle \chi_4 \rangle \sim \langle \chi_1 \rangle$ ,  $\langle \chi_2 \rangle \gg \langle \chi_5 \rangle$ ,  $\langle \chi_3 \rangle \gg \langle \chi_6 \rangle$ ,  $\langle \chi_2 \rangle \gg \langle \chi_1 \rangle$ , and  $\langle \chi_6 \rangle \gg \langle \chi_5 \rangle \gg \langle \chi_4 \rangle$ .

To account for the neutrino masses, an additional gauge singlet scalar field  $\chi_7$  is added, and neutrino mass patterns are recovered through the type-I seesaw mechanism. The HVM framework, however, does not offer any explanation to the leptonic mixing angles. This problem is addressed by a standard HVM (SHVM) [5], which provides remarkably precise predictions of leptonic mixing angles by linking them to the Cabibbo angle in the form of inequalities.

The realization of SHVM involves enlarging the HVM framework by extending the SM through the generic  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  symmetry. To produce the flavour structure of the SM assuming normal hierarchy for neutrino mass patterns, one must ensure that  $N = 2$ ,  $M \geq 3$ , and  $P \geq 14$  [5]. We take the  $\mathcal{Z}_2 \times \mathcal{Z}_4 \times \mathcal{Z}_{14}$  symmetry, and the charges to different SM fermionic and scalar fields under this symmetry are assigned, as shown in Table 5.

Fields	$\mathcal{Z}_2$	$\mathcal{Z}_4$	$\mathcal{Z}_{14}$	Fields	$\mathcal{Z}_2$	$\mathcal{Z}_4$	$\mathcal{Z}_{14}$	Fields	$\mathcal{Z}_2$	$\mathcal{Z}_4$	$\mathcal{Z}_{14}$	Fields	$\mathcal{Z}_2$	$\mathcal{Z}_4$	$\mathcal{Z}_{14}$
$u_R$	-	1	$\omega^{11}$	$d_R, s_R, b_R$	+	1	$\omega^{12}$	$\psi_{L_3}^q$	+	1	$\omega^2$	$\tau_R$	+	$\omega^3$	$\omega$
$c_R$	+	1	$\omega^6$	$\chi_4$	+	1	$\omega^{13}$	$\psi_{L_1}^l$	+	$\omega^3$	$\omega^{12}$	$\nu_{eR}$	+	$\omega$	$\omega^8$
$t_R$	+	1	$\omega^4$	$\chi_5$	+	1	$\omega^{11}$	$\psi_{L_2}^l$	+	$\omega^3$	$\omega^{10}$	$\nu_{\mu R}$	-	$\omega$	$\omega^3$
$\chi_1$	-	1	$\omega^2$	$\chi_6$	+	1	$\omega^{10}$	$\psi_{L_3}^l$	+	$\omega^3$	$\omega^6$	$\nu_{\tau R}$	-	$\omega$	$\omega^3$
$\chi_2$	+	1	$\omega^5$	$\psi_{L_1}^q$	+	1	$\omega^{13}$	$e_R$	-	$\omega^3$	$\omega^{10}$	$\chi_7$	-	$\omega^2$	$\omega^8$
$\chi_3$	+	1	$\omega^2$	$\psi_{L_2}^q$	+	1	$\omega$	$\mu_R$	+	$\omega^3$	$\omega^{13}$	$\phi$	+	1	1

TABLE 5: Charges assigned to SM fermions and scalar fields under  $\mathcal{Z}_2$ ,  $\mathcal{Z}_4$  and  $\mathcal{Z}_{14}$  symmetries, considering normal mass ordering for neutrinos.

The mass patterns of charged fermions are obtained through dimension-5 operators in the Lagrangian given in equation (18), considering the VEVs pattern of  $\chi_i$  fields as discussed above. The corresponding mass matrices are,

$$\mathcal{M}_U = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^u \epsilon_1 & 0 & y_{13}^u \epsilon_2 \\ 0 & y_{22}^u \epsilon_2 & y_{23}^u \epsilon_5 \\ 0 & y_{32}^u \epsilon_6 & y_{33}^u \epsilon_3 \end{pmatrix}, \quad \mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^d \epsilon_4 & y_{12}^d \epsilon_4 & y_{13}^d \epsilon_4 \\ y_{21}^d \epsilon_5 & y_{22}^d \epsilon_5 & y_{23}^d \epsilon_5 \\ y_{31}^d \epsilon_6 & y_{32}^d \epsilon_6 & y_{33}^d \epsilon_6 \end{pmatrix}, \quad \mathcal{M}_\ell = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^\ell \epsilon_1 & y_{12}^\ell \epsilon_4 & y_{13}^\ell \epsilon_5 \\ 0 & y_{22}^\ell \epsilon_5 & y_{23}^\ell \epsilon_2 \\ 0 & 0 & y_{33}^\ell \epsilon_2 \end{pmatrix}, \quad (19)$$

The masses of the charged fermions are retained as [5],

$$\begin{aligned} m_t &= |y_{33}^u| \epsilon_3 v / \sqrt{2}, \quad m_c = \left| y_{22}^u \epsilon_2 - \frac{y_{23}^u y_{32}^u \epsilon_5 \epsilon_6}{y_{33}^u \epsilon_3} \right| v / \sqrt{2}, \quad m_u = |y_{11}^u| \epsilon_1 v / \sqrt{2}, \\ m_b &\approx |y_{33}^d| \epsilon_6 v / \sqrt{2}, \quad m_s \approx \left| y_{22}^d - \frac{y_{23}^d y_{32}^d}{y_{33}^d} \right| \epsilon_5 v / \sqrt{2}, \\ m_d &\approx \left| y_{11}^d - \frac{y_{12}^d y_{21}^d}{y_{22}^d - \frac{y_{23}^d y_{32}^d}{y_{33}^d}} - \frac{y_{13}^d (y_{31}^d y_{22}^d - y_{21}^d y_{32}^d) - y_{31}^d y_{12}^d y_{23}^d}{\left( y_{22}^d - \frac{y_{23}^d y_{32}^d}{y_{33}^d} \right) y_{33}^d} \right| \epsilon_4 v / \sqrt{2}, \\ m_\tau &\approx |y_{33}^\ell| \epsilon_2 v / \sqrt{2}, \quad m_\mu \approx |y_{22}^\ell| \epsilon_5 v / \sqrt{2}, \quad m_e = |y_{11}^\ell| \epsilon_1 v / \sqrt{2}. \end{aligned} \quad (20)$$

The quark mixing angles are obtained as,

$$\sin \theta_{12} \simeq \left| \frac{y_{12}^d \epsilon_4}{y_{22}^d \epsilon_5} \right| = \frac{\epsilon_4}{\epsilon_5}, \quad \sin \theta_{23} \simeq \left| \frac{y_{23}^d \epsilon_5}{y_{33}^d \epsilon_6} - \frac{y_{23}^u \epsilon_5}{y_{33}^u \epsilon_3} \right| \approx \left| \frac{y_{23}^d}{y_{33}^d} \right| \sin \theta_{12}^2, \quad (21)$$

$$\sin \theta_{13} \simeq \left| \frac{y_{13}^d \epsilon_4}{y_{33}^d \epsilon_6} - \frac{y_{13}^u \epsilon_2}{y_{33}^u \epsilon_3} \right| \geq \left| \frac{y_{13}^d}{y_{33}^d} \right| \sin \theta_{12}^3 - \left| \frac{y_{13}^u}{y_{33}^u} \right| \frac{m_c}{m_t} \approx \sin \theta_{12}^3 - 2 \frac{m_c}{m_t}, \quad (22)$$

where  $\epsilon_i = \frac{\langle \chi_i \rangle}{\Lambda}$ ,  $\epsilon_i < 1$ , and we made the assumptions that  $\epsilon_4 / \epsilon_5 = \sin \theta_{12} = 0.225$ ,  $\epsilon_5 / \epsilon_6 = \sin \theta_{23}$ ,  $|y_{13}^d / y_{33}^d| \approx 1$ , and  $|y_{13}^u / y_{33}^u| \approx 2$ . Consequently, the SHVM predicts the quark mixing angle  $\sin \theta_{13}$  to be approximately  $\sin \theta_{12}^3 - 2(m_c / m_t)$ , aligning well with its experimentally observed value.

#### 4.1. Neutrino Masses and Leptonic Mixing Parameters

To recover neutrino masses, we add three right-handed neutrinos  $\nu_{eR}$ ,  $\nu_{\mu R}$ ,  $\nu_{\tau R}$  to the SM and write the corresponding Yukawa Lagrangian as,

$$-\mathcal{L}_{\text{Yukawa}}^{\nu} = y_{ij}^{\nu} \bar{\psi}_{L_i}^{\ell} \tilde{\phi}^{\nu} \nu_{f_R} \left[ \frac{\chi_i \chi_j \text{ (or } \chi_i \chi_j^{\dagger})}{\Lambda^2} \right] + \text{H.c.} \quad (23)$$

The Dirac mass matrix turn out to be,

$$\mathcal{M}_D = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{11}^{\nu} \epsilon_1 \epsilon_7 & y_{12}^{\nu} \epsilon_4 \epsilon_7 & y_{13}^{\nu} \epsilon_4 \epsilon_7 \\ 0 & y_{22}^{\nu} \epsilon_4 \epsilon_7 & y_{23}^{\nu} \epsilon_4 \epsilon_7 \\ 0 & y_{32}^{\nu} \epsilon_5 \epsilon_7 & y_{33}^{\nu} \epsilon_5 \epsilon_7 \end{pmatrix}, \quad (24)$$

which results in following mass expressions for neutrinos,

$$m_3 \approx |y_{33}^{\nu}| \epsilon_5 \epsilon_7 v / \sqrt{2}, \quad m_2 \approx \left| y_{22}^{\nu} - \frac{y_{23}^{\nu} y_{32}^{\nu}}{y_{33}^{\nu}} \right| \epsilon_4 \epsilon_7 v / \sqrt{2}, \quad m_1 \approx |y_{11}^{\nu}| \epsilon_1 \epsilon_7 v / \sqrt{2}. \quad (25)$$

For the benchmark values of  $y_{ij}^{\nu}$  obtained through our numerical analysis [5], the masses of neutrinos turn out to have the values  $\{m_3, m_2, m_1\} = \{5.05 \times 10^{-2}, 8.67 \times 10^{-3}, 2.67 \times 10^{-4}\}$  eV. The predictions for leptonic mixing are pivotal in SHVM. Assuming all the couplings to be of order one, the leptonic mixing angles can be expressed in terms of the Cabibbo angle in the following way [5],

$$\begin{aligned} \sin \theta_{12}^{\ell} &\simeq \left| -\frac{y_{12}^{\nu}}{y_{22}^{\nu}} + \frac{y_{12}^{\ell} \epsilon_4}{y_{22}^{\ell} \epsilon_5} + \frac{y_{23}^{\ell*} y_{13}^{\nu} \epsilon_4}{y_{33}^{\ell} y_{33}^{\nu} \epsilon_5} \right| \geq \left| -\frac{y_{12}^{\nu}}{y_{22}^{\nu}} \right| - \left| \frac{y_{12}^{\ell}}{y_{22}^{\ell}} + \frac{y_{23}^{\ell*} y_{13}^{\nu}}{y_{33}^{\ell} y_{33}^{\nu}} \right| \frac{\epsilon_4}{\epsilon_5} \geq 1 - 2 \sin \theta_{12}, \\ \sin \theta_{23}^{\ell} &\simeq \left| \frac{y_{23}^{\ell}}{y_{33}^{\ell}} - \frac{y_{23}^{\nu} \epsilon_4}{y_{33}^{\nu} \epsilon_5} \right| \geq \left| \frac{y_{23}^{\ell}}{y_{33}^{\ell}} \right| - \left| \frac{y_{23}^{\nu}}{y_{33}^{\nu}} \right| \frac{\epsilon_4}{\epsilon_5} \geq 1 - \sin \theta_{12}, \\ \sin \theta_{13}^{\ell} &\simeq \left| -\frac{y_{13}^{\nu} \epsilon_4}{y_{33}^{\nu} \epsilon_5} + \frac{y_{13}^{\ell} \epsilon_5}{y_{33}^{\ell} \epsilon_2} \right| \geq \left| -\frac{y_{13}^{\nu}}{y_{33}^{\nu}} \right| \frac{\epsilon_4}{\epsilon_5} - \left| \frac{y_{13}^{\ell}}{y_{33}^{\ell}} \right| \frac{\epsilon_5}{\epsilon_2} \geq \sin \theta_{12} - \frac{m_s}{m_c}, \end{aligned} \quad (26)$$

where  $m_s/m_c = \epsilon_5/\epsilon_2$ . The latest measurement of Cabibbo angle yields  $\sin \theta_{12} = (0.225 \pm 0.00067)$  [15]. It is clear from above equations that  $\sin \theta_{12}^\ell$  as well as  $\sin \theta_{23}^\ell$  shares the same precision of the Cabibbo angle's experimental precision. Using the Cabibbo angle measurement, leptonic mixing angles are obtained as  $\sin \theta_{12}^\ell = 0.55 \pm 0.00134$  and  $\sin \theta_{23}^\ell = 0.775 \pm 0.00067$ .

Using running masses of strange and charm quarks at 1 TeV ( $m_c = 0.532_{-0.073}^{+0.074}$  GeV,  $m_s = 4.7_{-1.3}^{+1.4} \times 10^{-2}$  GeV) [16], we obtain  $\sin \theta_{13}^\ell = (0.1169 - -0.1509)$ . However, the experimental lower end excludes (0.1169–0.1413), and the theory rules out the upper end (0.1509–0.1550). Thus, the final conclusive range for  $\sin \theta_{13}^\ell$  is predicted to be 0.1413–0.1509. The upcoming neutrino experiments like DUNE, Hyper-Kamiokande, and JUNO may test these predictions [17].

The SHVM rules out the possibility of neutrino masses originating from Majorana-type neutrinos, as the charge assignment of SHVM forbids both the Majorana Lagrangian and the Weinberg operators [5]. Consequently, the SHVM predicts neutrinos to be of the Dirac-type [5].

## 5. THE DARK TECHNICOLOUR PARADIGM

A UV completion of the SHVM can be obtained through the dark-technicolour framework [6]. The dark-technicolour model consists of the symmetry  $\mathcal{S} = SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(N_{TC}) \times SU(N_{DTC}) \times SU(N_F)$  symmetry, where TC stands for technicolour, DTC is for the dark-technicolour, and F is a strong dynamics of vector-like fermions [4]. The TC quarks transform under the symmetry  $\mathcal{S}$  as [4],

$$T_L^i \equiv \begin{pmatrix} T_i \\ B_i \end{pmatrix}_L : (1, 2, 0, N_{TC}, 1, 1), T_{i,R} : (1, 1, 1, N_{TC}, 1, 1), \quad (27)$$

$$B_{i,R} : (1, 1, -1, N_{TC}, 1, 1),$$

where electric charges  $+\frac{1}{2}$  for  $T$  and  $-\frac{1}{2}$  for  $B$ .

The DTC dynamics has the following fermions,

$$D \equiv C_{i,L,R} : (1, 1, 1, 1, N_{DTC}, 1), \quad S_{i,L,R} : (1, 1, -1, 1, N_{DTC}, 1), \quad (28)$$

where electric charges of the quark  $C$  is  $+\frac{1}{2}$ , and that of the  $S$  is  $-\frac{1}{2}$ .

The vector-like fermions of  $SU(N_F)$  dynamics transform as,

$$F_{L,R} \equiv U_{L,R}^i \equiv \left(3, 1, \frac{4}{3}, 1, 1, N_F\right), \quad D_{L,R}^i \equiv \left(3, 1, -\frac{2}{3}, 1, 1, N_F\right), \quad (29)$$

$$N_{L,R}^i \equiv (1, 1, 0, 1, 1, N_F), \quad E_{L,R}^i \equiv (1, 1, -2, 1, 1, N_F).$$

There are three axial  $U(1)_{A}^{TC,DTC,F}$  symmetries in the DTC paradigm which are broken as  $U(1)_{A}^{TC,DTC,F} \rightarrow \mathcal{Z}_{2K_{TC,DTC,F}}$  producing the generic  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  flavour symmetry, where  $N = 2K_{TC}$ ,  $M = 2K_{DTC}$  and  $P = 2K_F$  [18].

The mass of a charged fermion is originated from the interactions given in Figure 3 and turns out to be [6],

$$m_f = y_f \frac{\Lambda_{TC}^3}{\Lambda_{ETC}^2} \frac{1}{\Lambda_F} \frac{\Lambda_{DTC}^{n_i+1}}{\Lambda_{EDTC}^{n_i}} \exp(n_i k), \quad (30)$$

where  $n_i = 2, 4, \dots, 2n$  represents the number of fermions in a multi-fermion chiral condensate playing the role of the VEV  $\langle \chi_i \rangle$  in the SHVM [4]. The multi-fermion condensate can be parametrized as [19],

$$\langle (\bar{\psi}_L \psi_R)^n \rangle \sim (\Lambda \exp(k\Delta\chi))^{3n}, \quad (31)$$

where  $\Delta\chi$  shows the chirality of an operator,  $k$  denotes a constant, and  $\Lambda$  is the scale of the gauge theory. From this, we conclude that,

$$\epsilon_i = \frac{1}{\Lambda_F} \frac{\Lambda_{DTC}^{n_i+1}}{\Lambda_{EDTC}^{n_i}} \exp(n_i k). \quad (32)$$

The neutrino masses are recovered by the generic interactions given in Figure 4, where it is assumed that the TC and DTC sectors are accommodated in a GUT theory where the interactions between the  $F_L$  and  $F_R$  fermions are mediated by the gauge bosons of the GUT. The VEV  $\langle \chi_7 \rangle$  corresponds to the chiral condensate  $\langle \bar{F}_L F_R \rangle$ . The masses of neutrinos become,

$$m_\nu = y_f \frac{\Lambda_{TC}^3}{\Lambda_{ETC}^2} \frac{1}{\Lambda_F} \frac{\Lambda_{DTC}^{n_i+1}}{\Lambda_{EDTC}^{n_i}} \exp(n_i k) \frac{1}{\Lambda_F} \frac{\Lambda_F^3}{\Lambda_{GUT}^2} \exp(2k). \quad (33)$$

For more details, see [6].

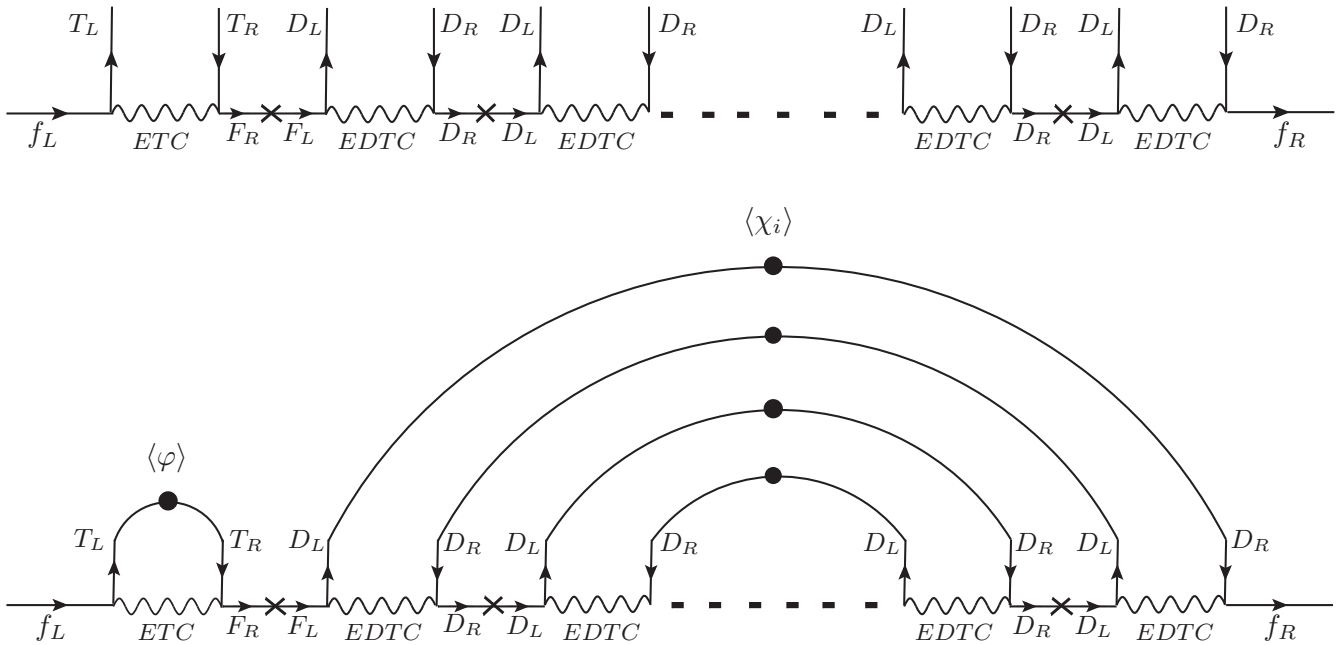


FIGURE 3: Feynman diagrams corresponding to charged fermion masses in the dark-technicolour paradigm. The diagram at the top depicts generic interactions of SM, TC, F, and DTC fermions, while diagram at the bottom shows the formation of fermionic condensates resulting in masses for charged SM fermions.

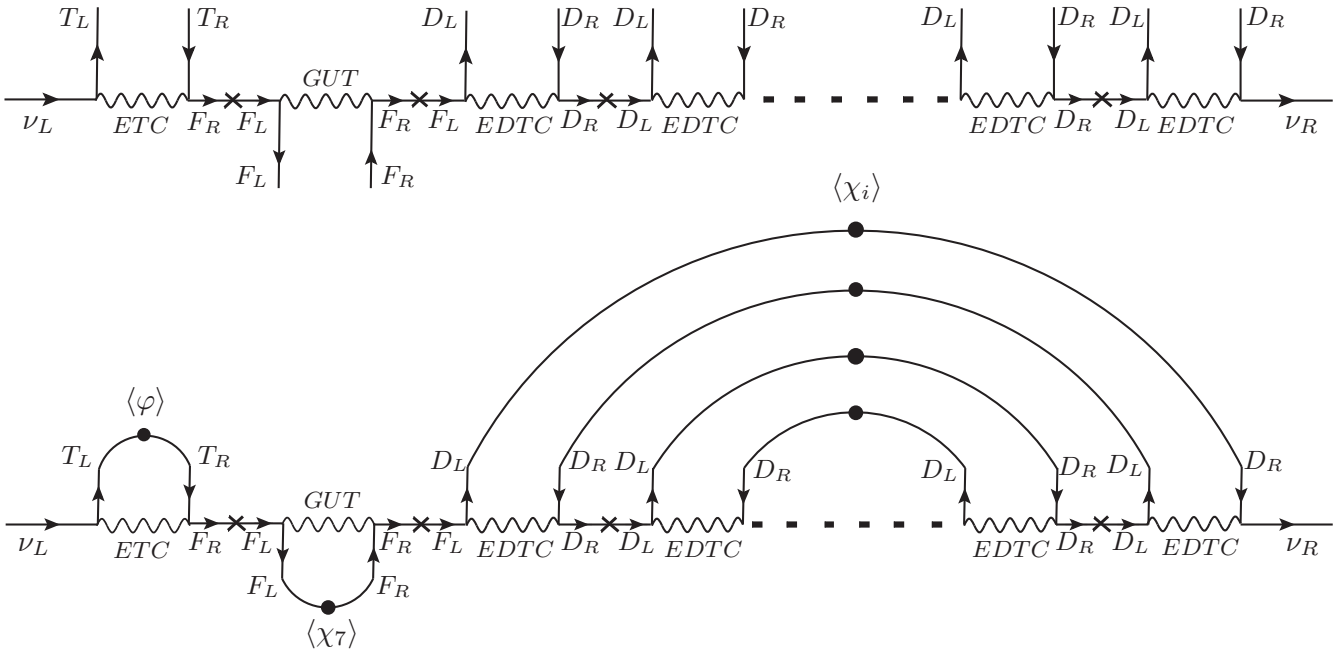


FIGURE 4: Feynman diagrams corresponding to the masses of neutrinos in the dark-technicolour paradigm. The diagram at the top shows generic interactions among the SM, TC, DTC gauge sectors, while the diagram at the bottom represents the formation of fermionic condensates leading to neutrino masses.

### 6. SUMMARY

In this work, we have discussed new approaches to address the flavour problem and dark matter together. The flavour bounds on the models based on  $Z_N \times Z_M$  flavour symmetries are extremely stringent in the phase-II of the LHCb, and most of the parameter space is ruled out.



These new frameworks provides origins of the flavour problem and dark matter based on discrete  $\mathcal{Z}_N \times \mathcal{Z}_M \times \mathcal{Z}_P$  flavour symmetries. Origin of these symmetries may lie in a technicolour paradigm. We notice that these new frameworks result in a new class of dark matter referred to as the flavonic dark matter.

## ACKNOWLEDGMENTS

This work is supported by the Council of Science and Technology, Govt. of Uttar Pradesh, India through the project ‘‘A new paradigm for flavour problem’’ no. CST/D-1301, and Science and Engineering Research Board, Department of Science and Technology, Government of India through the project ‘‘Higgs Physics within and beyond the Standard Model’’ no. CRG/2022/003237. Neelam Singh acknowledges the support through the INSPIRE fellowship by the Department of Science and Technology, Government of India.

## References

- [1] G. Abbas, R. Adhikari, E. J. Chun, and N. Singh, ‘‘The problem of flavour,’’ [arXiv:2308.14811 [hep-ph]].
- [2] G. Abbas, ‘‘Solving the fermionic mass hierarchy of the standard model,’’ *Int. J. Mod. Phys. A* **34** (2019) no.20, 1950104 doi:10.1142/S0217751X19501045 [arXiv:1712.08052 [hep-ph]].
- [3] G. Abbas, ‘‘A new solution of the fermionic mass hierarchy of the standard model,’’ *Int. J. Mod. Phys. A* **36**, no.18, 2150090 (2021) doi:10.1142/S0217751X21500901 [arXiv:1807.05683 [hep-ph]].
- [4] G. Abbas, ‘‘Origin of the VEVs hierarchy,’’ *Int. J. Mod. Phys. A* **37**, no.11n12, 2250056 (2022) doi:10.1142/S0217751X22500567 [arXiv:2012.11283 [hep-ph]].
- [5] G. Abbas, ‘‘The standard HVM,’’ [arXiv:2310.12915 [hep-ph]].
- [6] G. Abbas and N. Singh, ‘‘Dark-technicolour at low scale,’’ [arXiv:2312.16532 [hep-ph]].
- [7] G. Abbas, R. Adhikari, and E. J. Chun, ‘‘Flavonic dark matter,’’ *Phys. Rev. D* **108**, no.11, 115035 (2023) doi:10.1103/PhysRevD.108.115035 [arXiv:2303.10125 [hep-ph]].
- [8] C. D. Froggatt and H. B. Nielsen, ‘‘Hierarchy of Quark Masses, Cabibbo Angles and CP Violation,’’ *Nucl. Phys. B* **147**, 277–298 (1979) doi:10.1016/0550-3213(79)90316-X
- [9] G. Abbas, V. Singh, N. Singh, and R. Sain, ‘‘Flavour bounds on the flavon of a minimal and a non-minimal  $\mathcal{Z}_2 \times \mathcal{Z}_N$  symmetry,’’ *Eur. Phys. J. C* **83**, no.4, 305 (2023) doi:10.1140/epjc/s10052-023-11471-5 [arXiv:2208.03733 [hep-ph]].
- [10] A. Rasin, ‘‘Hierarchical quark mass matrices,’’ *Phys. Rev. D* **58**, 096012 (1998) doi:10.1103/PhysRevD.58.096012 [arXiv:hep-ph/9802356 [hep-ph]].
- [11] F. Calore, A. Dekker, P. D. Serpico, and T. Siebert, *Mon. Not. Roy. Astron. Soc.* **520** (2023) 3, 4167–4172 [arXiv:2209.06299 [hep-ph]].
- [12] L. Brouwer et al. [DMRadio], ‘‘Proposal for a definitive search for GUT-scale QCD axions,’’ *Phys. Rev. D* **106** (2022) no.11, 112003 doi:10.1103/PhysRevD.106.112003 [arXiv:2203.11246 [hep-ex]].
- [13] C. Thorpe-Morgan, D. Malyshev, A. Santangelo, J. Jochum, B. Jager, M. Sasaki, and S. Saeedi, ‘‘THESEUS insights into axionlike particles, dark photon, and sterile neutrino dark matter,’’ *Phys. Rev. D* **102** (2020) no.12, 123003 doi:10.1103/PhysRevD.102.123003 [arXiv:2008.08306 [astro-ph.HE]].
- [14] C. Antel, et al., ‘‘Feebly Interacting Particles: FIPs 2022 workshop report,’’ [arXiv:2305.01715 [hep-ph]].
- [15] R. L. Workman et al. (Particle Data Group), *Prog. Theor. Exp. Phys.* **2022**, 083C01 (2022)
- [16] Z. z. Xing, H. Zhang, and S. Zhou, ‘‘Updated Values of Running Quark and Lepton Masses,’’ *Phys. Rev. D* **77**, 113016 (2008) doi:10.1103/PhysRevD.77.113016 [arXiv:0712.1419 [hep-ph]].
- [17] P. Huber, K. Scholberg, E. Worcester, J. Asaadi, A. B. Balantekin, N. Bowden, P. Coloma, P. B. Denton, A. de Gouvˆea and L. Fields, et al., ‘‘Snowmass Neutrino Frontier Report,’’ [arXiv:2211.08641 [hep-ex]].
- [18] H. Harari and N. Seiberg, ‘‘Generation Labels in Composite Models for Quarks and Leptons,’’ *Phys. Lett. B* **102**, 263–266 (1981) doi:10.1016/0370-2693(81)90871-6
- [19] K. I. Aoki and M. Bando, ‘‘Hierarchical Multi - Fermion Condensation in Extended Mac Scenario,’’ *Prog. Theor. Phys.* **70**, 272 (1983) doi:10.1143/PTP.70.272