B-Flavour and a_μ Anomalies with S_1 Leptoquark in $SO(10)$ Grand Unification

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Abstract

The relatively long-existing *B*-flavour anomalies at the LHC searches have caused excitement in the last decade as possible indications of new physics beyond the Standard Model. Even though the recent news that one of these anomalies (namely the $R_{K^{(*)}}$ anomaly) appears to have disappeared from the data has caused some readjustments in our expectations, the still-existing anomalies in experiments remain to provide some semblance of anticipation for new physics. Among these are the *B*-decay anomaly called $R_{D^{(*)}}$ and the almost two-decade-old measurement problem of the muon magnetic moment (a_μ). In this conference paper, I will discuss the $S_1(3, 1, -1/3)$ leptoquark solution of these anomalies in $SO(10)$ grand unification.

1. INTRODUCTION AND OUTLOOK

[1]The LHC discovered the Higgs boson as its main objective, which was a great success for the high-energy physics community. On the other hand, there was a high expectation of discovering new physics, based on the paradigms that have led to the remarkable success of the SM over the years. Even though there has been no confirmed discovery yet at the LHC, there are several reported anomalies, namely *B*-decay anomalies (see [3, 4] for recent developments), which have been somewhat persistent over the years. If confirmed, these could be an indication of new physics. In addition, the Muon *g* − 2 Collaboration at Fermilab, relatively recently, announced new results on the anomalous magnetic moment of the muon [5], reporting the most precise measurement of the well-known *g* − 2 (or a_μ) anomaly [6].[2]

In this conference paper, I will focus on the $S_1(3, 1, -1/3)$ scalar leptoquark explanation of *B*-decay and *g* − 2 anomalies in $SO(10)$ grand unified theory (GUT) framework. The most significant *B* anomalies have been observed in the $R_{D^{(*)}}$, whose experimental values are given by the Heavy Flavor Averaging Group [8] as

$$R_D = 0.340 \pm 0.027(\text{stat}) \pm 0.013(\text{syst}),$$
$$R_{D^*} = 0.295 \pm 0.011(\text{stat}) \pm 0.008(\text{syst}),$$
(1)

which are in 3.2σ excess of the SM predictions, $R_{D^{(*)}}^{\text{SM}} = 0.299$ (0.258). The $R_{K^{(*)}}$ anomaly, previously reported as a 3.1σ discrepancy by the LHCb collaboration [9], seems to fade away with the new data [10]. Another persisting anomaly that has been around for almost two decades is in the measurement of the magnetic moment of the muon [6], a_μ, whose current average reads [5]

$$a_\mu = (116\,592\,061 \pm 41) \times 10^{-11},$$
(2)

which deviates at 4.2σ from $a_\mu^{\text{SM}} = (116\,591\,810 \pm 43) \times 10^{-11}$, with $\Delta a_\mu = (25.1 \pm 5.9) \times 10^{-10}$ [11].

A common approach to address these anomalies involves the existence of leptoquarks at the TeV scale. These states possess quantum numbers that allow them to couple both leptons and quarks and they often exist in GUTs in vector and scalar forms. In the unification framework, we are interested in here, it appears to be less convenient to follow the vector leptoquark route since they arise as the gauge bosons of $SU(4)$, and it is difficult to locate them in the TeV scale spectrum due to the phenomenological constraints. For instance, the rare decays $K_L \rightarrow \mu^\pm e^\mp$ bounds the mass of the vector leptoquark from below as $\sim$1000 TeV [12] (but see [13, 14, 15, 16] for some recent developments). Thus, we are interested in scalar leptoquarks here. Since the $R_{K^{(*)}}$ anomaly appears to disappear with the recent data [10], while the $R_{D^{(*)}}$ anomaly persists, the single leptoquark solution of S_1 resurrects as a single particle solution for this and *g* − 2 anomalies.

Since S_1, in principle could mediate proton decay, it has been a common practice in the GUT literature to conjecture S_1 heavy, preferably close to the GUT scale, so that these effects are suppressed in such a way to be consistent with the proton decay constraints. Since S_1 leptoquark resides in the same parent multiplet as the Higgs doublet in many models such as the ones based on

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[1]This talk is based on the works done in collaboration with Tanumoy Mandal, Subhadip Mitra, and Shoaib Munir [1, 2].

[2]Most recently, the CDF collaboration, at Fermilab, reported the *W* boson mass anomaly [7], although the result requires further confirmation by other groups.

$SU(5)$ or $SO(10)$ gauge symmetries, keeping S_1 heavy while getting a TeV scale Higgs is the infamous issue known as the doublet-triplet splitting problem in supersymmetric theories and/or GUTs. But of course, there is no need to be overly dismissive regarding light S_1 since there could be numerous ways to deal with proton decay including symmetry mechanisms such as the utilization of Peccei-Quinn (PQ) symmetry [17, 18], other $U(1)$ symmetries such as the one discussed in [19], or a discrete symmetry similar to the one considered in [20], the last of which will be used in this paper. Moreover, the corresponding operators could also be suppressed by a specific mechanism such as the one discussed in [21]. Furthermore, such operators could also be forbidden for geometrical reasons as in the Pati-Salam models based on Non-commutative geometry [22].

In fact, it is conceivable in the GUT framework to anticipate TeV scale degrees of freedom, companion to the SM Higgs doublet in the parent multiplet. This would be a single S_1 for an $SO(10)$ model with a real scalar multiplet $\mathbf{10}_H$ [1], whereas 2 S_1's and another Higgs doublet in the case of a complex $\mathbf{10}_H$ [2]. The tree-level mass terms are parametrized with the same parameters up of $O(1)$ coefficients due to the potential terms (see the mass matrices given in [23]). So, it is not difficult to imagine that whatever fine-tunning (or some mechanism) keeps the SM Higgs at the low energies brings the rest of the $\mathbf{10}_H$ with it.

Therefore, detecting a S_1 leptoquark at the TeV scale could be interpreted as evidence towards unification since it is hard to imagine another reason for a single S_1 to appear with the SM Higgs around the electroweak scale (EW) other than them being in the same parent multiplet. This, of course, avoids the problem of the infamous double-triplet mass splitting. Yet, the fine-tuning problem is still there to get these EW-TeV scale particle masses while there is a GUT scale contribution due to the VEV of the scalar that breaks the GUT symmetry. But from this point of view, even the SM Higgs being at the EW scale would be troublesome in the GUT framework. Consequently, detecting a scalar like a S_1 close to the TeV scale would strengthen our suspicion that there is something deep we don't understand about naturalness.

2. THE $SO(10)$ MODEL

Each family of SM fermions (plus a right-handed (RH) neutrino) is put in the spinor representation $\mathbf{16}$ of the $SO(10)$ group. Based on the relation $\mathbf{16} \otimes \mathbf{16} = \mathbf{10} \oplus \mathbf{120} \oplus \mathbf{126}$, the scalar content for the Yukawa sector of the model should be selected in a combination of representation on the right-hand side. As mentioned above, $\mathbf{10}_H$ contains the SM Higgs but by itself, this multiplet is not enough to give a realistic Yukawa sector regarding GUT scale fermion mass relations [18]. The appropriate scalar content depends on the scalar $\mathbf{10}$ being chosen as complex or real [23, 24].

In [1, 2], we investigated the real and complex scalar $\mathbf{10}_H$ cases, respectively. In the former, as noted above, we have a single S_1 at the TeV scale, which is enough to address $R_{D^{(*)}}$ and a_μ anomalies, whereas in the complex case we have a version of 2HDM with 2 S_1's, which yields a richer phenomenology, noting that 2HDMs have their own motivation as new physics [25]. Here, we continue with the real $\mathbf{10}_H$ case. We adopt the $SO(10)$ scalar field content of [24] to be consistent with the realistic Yukawa sector they have achieved. The scalar sector consists of a real $\mathbf{10}_H$, a real $\mathbf{120}_H$, and a complex $\mathbf{126}_H$, as well as a $\mathbf{54}_H$ to break the $SO(10)$ symmetry, as noted below.

A common breaking sequence of the gauge symmetry in this framework is schematically given as

$$SO(10) \xrightarrow[\langle \mathbf{54}_H \rangle]{M_U} G_{422D} \xrightarrow[\langle \mathbf{126}_H \rangle]{M_{\text{PS}}} G_{321} \text{ (SM)} \xrightarrow[\langle \mathbf{10}_H \rangle]{M_Z} G_{31}, \tag{3}$$

where G_{422D}, G_{321}, and G_{31} denote the left-right (LR) symmetric Pati-Salam (PS), SM, and the post-electroweak-symmetry-breaking gauge symmetries. The LR symmetry in the Pati-Salam phase, denoted by the symbol D, is optional and one can choose the route without it, in which case the breaking of $SO(10)$ should be realized with $\mathbf{210}_H$, instead of $\mathbf{54}_H$, since it contains the appropriate Pati-Salam singlet. The second part of the symmetry-breaking, from PS to SM, is achieved by the SM singlet contained in $\Delta_R(10, 1, 3)_{422}$ of $\mathbf{126}_H$, acquiring the vacuum expectation value. The scalar fields active in the corresponding energy intervals are given in Table 1. Note that in addition to the bidoublet ϕ and the color sextet Φ, originating from $\mathbf{10}_H$, we also have a second bidoublet and a Σ field, coming from $\mathbf{120}_H$, and a second Σ field, orignating from $\mathbf{126}_H$, at M_{PS} for the sake of a viable Yukawa sector [24]. The energy scales and the unification coupling, shown in Figure 1, are found as [1]

$$\log_{10}\left(\frac{M_U}{\text{GeV}}\right) = 15.6, \quad \log_{10}\left(\frac{M_{\text{PS}}}{\text{GeV}}\right) = 13.7, \quad \alpha_U^{-1} = 35.4. \tag{4}$$

As explained below, we forbid the proton-decay-mediating couplings of S_1 by a discrete symmetry, but we do not make any assumptions regarding the other potentially dangerous operators. The most stringent bound on the lifetime of the proton comes from the mode $p \to e^+ \pi^0$, and is $\tau_p > 1.6 \times 10^{34}$ years [26]. For the proton decay modes that are mediated by the super-heavy gauge bosons, which reside in the adjoint representation of $SO(10)$ $\mathbf{45}$, considering that $\tau_p \sim M_U^4/m_p^5 \alpha_U^2$ [27], we obtain $M_U \gtrsim 10^{15.9}$ GeV, which is consistent with the value given in equation (4) up to a factor of 2. Additionally, there exist proton-decay-mediating color triplets at M_{PS}. From a naive analysis [28], it can be shown that the current bounds on the proton lifetime require $M_{\text{PS}} \gtrsim 10^{11}$ GeV, again consistent with equation (4).
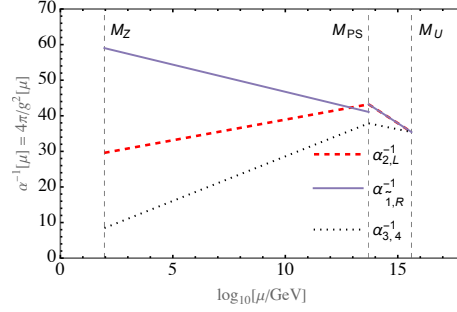
2.1. Low Energy Phenomenology

The interaction terms in the low energy Lagrangian relevant for the anomalies are given as

$$\mathcal{L} \supset \left[\lambda_{ij}^L \bar{Q}_i^c (i\tau_2) L_j + \lambda_{ij}^R \bar{u}_i^c \ell_{Rj} \right] S_1^\dagger + \text{H.c.}, \tag{5}$$

Interval	Scalar content for model
$M_U - M_{\mathrm{PS}}$	$\phi(1,2,2) \times 2$, $\Phi(6,1,1)$, $\Sigma(15,2,2) \times 2$, $\Delta_R(\overline{10},1,3)$, $\Delta_L(10,3,1)$
$M_{\mathrm{PS}} - M_Z$	$H\left(1,2,\frac{1}{2}\right)$, $S_1\left(3,1,-\frac{1}{3}\right)$

TABLE 1: The scalar content of the model.

FIGURE 1: Adapted from [1]. Running of the gauge couplings. Note that $\alpha_{\tilde{1}}^{-1} \equiv \frac{3}{5}\alpha_1^{-1}$.

where Q_i and L_i denote the ith-generation quark and lepton doublets, respectively, and $\lambda_{ij}^{L,R}$ represents the coupling of S_1 with a charge-conjugate quark of ith generation and a lepton of j-th generation with chirality L, R. We prevent the diquark couplings, which would lead to proton decay at the tree level, by imposing an, admittedly ad-hoc, discrete symmetry assumed to emerge below the Pati-Salam breaking scale. Under this symmetry $(q, l, S_1) \to (\pm q, \mp l, -S_1)$, where q (l) denotes any quark (lepton) and where the rest of the particle content does not transform.

The ratios $r_{D^{(*)}} = R_{D^{(*)}}/R_{D^{(*)}}^{\mathrm{SM}}$ are given as [29]

$$r_D \equiv \frac{R_D}{R_D^{\mathrm{SM}}} \approx |1 + \mathcal{C}_{V_L}|^2 + 1.02\,|\mathcal{C}_{S_L}|^2 + 0.9\,|\mathcal{C}_{T_L}|^2 + 1.49\,\mathrm{Re}\left[(1 + \mathcal{C}_{V_L})\,\mathcal{C}_{S_L}^*\right] + 1.14\,\mathrm{Re}\left[(1 + \mathcal{C}_{V_L})\,\mathcal{C}_{T_L}^*\right], \tag{6}$$

$$r_{D^*} \equiv \frac{R_{D^*}}{R_D^{\mathrm{SM}}} \approx |1 + \mathcal{C}_{V_L}|^2 + 0.04\,|\mathcal{C}_{S_L}|^2 + 16.07\,|\mathcal{C}_{T_L}|^2 - 0.11\,\mathrm{Re}\left[(1 + \mathcal{C}_{V_L})\,\mathcal{C}_{S_L}^*\right] - 5.12\,\mathrm{Re}\left[(1 + \mathcal{C}_{V_L})\,\mathcal{C}_{T_L}^*\right], \tag{7}$$

where

$$\begin{aligned}
\mathcal{C}_{V_L} &= \frac{1}{2\sqrt{2}G_F V_{cb}}\frac{\lambda_{23}^{L*}\lambda_{33}^{L}}{2M_{S_1}^2}, \\
\mathcal{C}_{S_L} &= -\frac{1}{2\sqrt{2}G_F V_{cb}}\frac{\lambda_{33}^{L}\lambda_{23}^{R}}{2M_{S_1}^2}, \\
\mathcal{C}_{T_L} &= -\frac{1}{4}\mathcal{C}_{S_L},
\end{aligned} \tag{8}$$

are the Wilson coefficients corresponding to operators

$$\begin{aligned}
\mathcal{O}_{V_L} &= \left[\bar{c}\gamma^\mu P_L b\right]\left[\bar{\tau}\gamma_\mu P_L \nu\right], \\
\mathcal{O}_{S_L} &= \left[\bar{c}P_L b\right]\left[\bar{\tau}P_L \nu\right], \\
\mathcal{O}_{T_L} &= \left[\bar{c}\sigma^{\mu\nu} P_L b\right]\left[\bar{\tau}\sigma_{\mu\nu} P_L \nu\right].
\end{aligned} \tag{9}$$

The contribution of S_1 to a_μ can be approximated by [30]

$$\Delta a_\mu \simeq -\frac{N_c}{8\pi^2}\frac{m_t m_\mu}{M_{S_1}^2}V_{tb}\lambda_{32}^{L}\lambda_{32}^{R}\left[\frac{7}{6} + \frac{2}{3}\log x_t\right], \tag{10}$$

where m_t (μ_μ) is the top (muon) mass, $x_t = m_t^2/M_{S_1}^2$, and V_{tb} is the relevant CKM matrix element.

In [1], based on the $SO(10)$ model with a real $\mathbf{10}_H$, we investigated the parameter space for a single S_1, namely $(\lambda_{33}^{L}, \lambda_{23}^{R}, \lambda_{23}^{L})$ and M_{S_1}, that explains the $R_{D^{(*)}}$ anomalies while taking into account the relevant flavour constraints on $F_L(D^*)$, $P_\tau(D^*)$, and $R_{K^{(*)}}^{\nu\nu}$ as well as the constraint coming from the $Z \to \tau\tau$ decay and the $\tau\tau$ resonance search data at the LHC. In [2], even though we

looked at the complex $\mathbf{10}_H$ case which has a richer particle spectrum at the TeV scale, the information we extracted on the couplings relevant to a_μ anomaly, namely $(\lambda_{32}^L, \lambda_{32}^R)$, is valid for the single S_1 case as well. Due to the limited space allotted for this article, we do not include all the plots here; the interested readers are referred to the papers above. In short, we found numerous points that were consistent with the anomalies in the perturbative range. The other question is whether these points can remain in the perturbative range $(-\sqrt{4\pi}, \sqrt{4\pi})$ at high energies, as required for consistency with our high-energy analysis. Indeed, some portion of the parameter space remains in the perturbative region as shown for some benchmark points in Figures 2(a) and 2(b). We also include an example in Figure 2(c), where the perturbativity limit is hit well below the unification scale. Note that we perform the Yukawa RG running by ignoring the changes expected at the intermediate symmetry-breaking scale since these effects are expected to be minor [28].
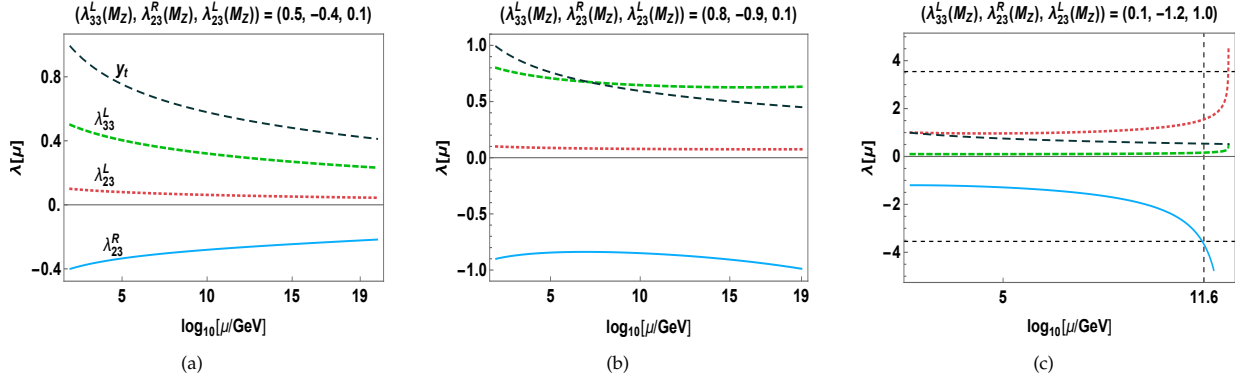

(a)

(b)

(c)

FIGURE 2: Taken from [1]. The behavior of Yukawa couplings with various benchmark values at the EW scale. The labels of the couplings are given in the first plot. The dashed horizontal lines denote the values of the assumed perturbativity bound, $\pm\sqrt{4\pi}$. The dashed vertical line in (c) denotes the energy scale at which this bound is first reached.

$(\lambda_{33}^L, \lambda_{23}^R, \lambda_{23}^L)$ Fermion masses/ratios	SM	BP1	BP2
m_t/m_b	75.24	75.32	75.97
m_τ/m_b	1.60	1.71	2.22
m_μ/m_s	4.34	4.38	4.38
m_e/m_d	0.390	0.395	0.395
m_t/GeV	81.12	81.34	85.14
m_c/GeV	0.261	0.261	0.281
$m_\mu/(10^{-3}\,\text{GeV})$	101.248	101.322	102.206
$m_e/(10^{-3}\,\text{GeV})$	0.480	0.480	0.484
$m_u/(10^{-3}\,\text{GeV})$	0.482	0.477	0.481

TABLE 2: Fermion masses at the unification scale for benchmark points (BPs) for $(\lambda_{33}^L(M_Z), \lambda_{23}^R(M_Z), \lambda_{23}^L(M_Z))$; BP1 = $(0.5, -0.4, 0.1)$ and BP2 = $(0.8, -0.9, 0.1)$. We also display the SM values at the unification scale.

We also give the fermion mass relations at the unification scale, $M_U = 4.0 \times 10^{15}\,\text{GeV}$, in Table 2 to demonstrate that the inclusion of S_1 in the low-energy particle spectrum does not lead to significant changes in the fermion mass values at the unification scale compared to the SM predictions.

Note that since the Yukawa analysis above are from [1], where we did not address the a_μ anomaly but only $R_{D^{(*)}}$, it only takes into account the constraints on $(\lambda_{33}^L, \lambda_{23}^R, \lambda_{23}^L)$. This should be revised with the information on $(\lambda_{32}^L, \lambda_{32}^R)$ from the a_μ data, which we leave for future work.

3. SUMMARY

In this conference paper, I discussed the possible explanation of the $R_{D^{(*)}}$ and $g-2$ anomalies via an $S_1(3, 1, -1/3)$ leptoquark in the $SO(10)$ GUT framework. I adopted and attempted to motivate the approach that $\mathbf{10}_H$ in the $SO(10)$ framework remains light altogether. In the case of a real $\mathbf{10}_H$, which is the main focus of this paper, the only extra degree of freedom at the TeV scale is just a single scalar leptoquark S_1, whereas in the complex case, we have a version of 2HDM with 2 S_1's. In [1, 2], we investigated the real and complex cases, respectively, and explored the corresponding parameter space that is consistent with all the relevant constraints. The gauge and Yukawa couplings appear to remain in the perturbative regime throughout the RG running for a portion

of the parameter space and the GUT scale fermion mass predictions are only slightly different than those of the SM. The proton decay is prevented by an imposed discrete symmetry that is assumed to emerge at low energies.

ACKNOWLEDGMENTS

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