Swampland Program, Extra Dimensions, and Supersymmetry Breaking

Luis A. Anchordoqui,[1,2,3] Ignatios Antoniadis,[4,5] and Dieter Lüst[6,7]

[1] *Department of Physics and Astronomy, Lehman College, City University of New York, NY 10468, USA*
[2] *Department of Physics, Graduate Center, City University of New York, NY 10016, USA*
[3] *Department of Astrophysics, American Museum of Natural History, NY 10024, USA*
[4] *Laboratoire de Physique Théorique et Hautes Énergies - LPTHE, Sorbonne Université, CNRS, 4 Place Jussieu, 75005 Paris, France*
[5] *Center for Cosmology and Particle Physics, Department of Physics, New York University, 726 Broadway, New York, NY 10003, USA*
[6] *Max-Planck-Institut für Physik, Werner–Heisenberg–Institut, 80805 München, Germany*
[7] *Arnold Sommerfeld Center for Theoretical Physics, Ludwig-Maximilians-Universität München, 80333 München, Germany*

Abstract

By combining swampland conjectures with observational data, it was recently suggested that the cosmological hierarchy problem (i.e., the smallness of the dark energy in Planck units) could be understood as an asymptotic limit in field space, corresponding to a decompactification of one extra (dark) dimension of a size in the micron range. In these Proceedings we examine the fundamental setting of this framework and discuss general aspects of the effective low energy theory inherited from properties of the overarching string theory. We then explore some novel phenomenology encompassing the dark dimension by looking at potential dark matter candidates, decoding neutrino masses, and digging into new cosmological phenomena.

Keywords: extra dimensions, swampland distance conjecture, primordial black holes, supersymmetry breaking
DOI: 10.31526/BSM-2023.11

1. INTRODUCTION

The challenge for a fundamental theory of nature is to describe both particle physics and cosmology. Accelerator experiments and cosmological observations provide complementary information to constrain the same theory. We have long known that only about 4% of the content of the universe is ordinary baryonic matter; the remainder is dark matter (∼22%) and dark energy (∼74%). The ΛCDM model, in which the expansion of the universe today is dominated by the cosmological constant Λ and cold dark matter (CDM), is the simplest model that provides a reasonably good account of all astronomical and cosmological observations [1].

The cosmological evolution is described by Einstein's equation,

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1}$$

where $R_{\mu\nu}$ and R are respectively the Ricci tensor and scalar, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the energy momentum tensor, and $G = 1/(8\pi M_p^2)$ is Newton's gravitational constant. The cosmological constant encapsulates two length scales: the size of the observable Universe $[\Lambda] = L^{-2}$ and of the dark energy $[\Lambda/G \times c^3/\hbar] = L^{-4}$. The observed value of the cosmological constant $\Lambda_{\rm obs} \simeq 0.74 \times 3H_0^2/c^2 \simeq 1.4 \times (10^{26}\,{\rm m})^{-2}$ gives a characteristic length of dark energy $\simeq 85\,\mu$m, where we have adopted the recent measurement of the Hubble constant $H_0 \simeq 73\,{\rm km/s/Mpc}$ by the HST + SH0ES team [2].

At currently achievable collider center-of-mass energies $\sqrt{s} \sim 14\,{\rm TeV}$ or, equivalently, at distance scales $<10^{-21}\,{\rm m}$, the Standard Model (SM) of strong and electroweak interactions, amended with appropriate neutrino masses, provides a successful and predictive theoretical description of all available data [1]. The experimental success of the SM can be considered as the triumph of the gauge symmetry principle to describe particle interactions. Its gauge structure is described by the symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, with electroweak symmetry breaking at an energy scale of $M_{ew} \sim$ TeV. On the grounds of this, the masses of the weak force carriers ($W^\pm$ and Z^0) are about 16 orders of magnitude smaller than M_p and so the weak force is 10^{24} times stronger than gravity.

A way to connect these hierarchies between particle physics and cosmology is via the size of extra dimensions which are necessary ingredients for consistency of string theory [3]. Indeed, if their size is large compared to the fundamental (string) length, the strength of gravitational interactions becomes strong at distances larger than the actual four-dimensional (4D) Planck length [4, 5]. As a result, the string scale is detached from the Planck mass consistently with all experimental bounds if the observable universe is localized in the large compact space [5].

In these Proceedings we summarize the state-of-the-art in this subject area, and discuss future research directions.

2. FOUNDATIONS OF THE DARK DIMENSION

The Swampland program seeks to understand which are the "good" low-energy EFTs that can couple to gravity consistently (e.g., the landscape of superstring theory vacua) and distinguish them from the "bad" ones that cannot [6]. In theory space, the frontier discerning the good theories from those downgraded to the swampland is drawn by a family of conjectures classifying the properties that an EFT should call for/avoid to enable a consistent completion into quantum gravity. These conjectures provide a bridge from quantum gravity to astrophysics, cosmology, and particle physics [7, 8, 9].

For example, the distance conjecture (DC) forecasts the appearance of infinite towers of states that become exponentially light and trigger the collapse of the EFT at infinite distance limits in moduli space [10]. Connected to the DC is the anti-de Sitter (AdS) distance conjecture, which correlates the dark energy density to the mass scale m characterizing the infinite tower of states, $m \sim |\Lambda|^\alpha$, as the negative AdS vacuum energy $\Lambda \to 0$, with α a positive constant of $\mathcal{O}(1)$ [11]. Besides, under the hypothesis that this scaling behavior holds in dS (or quasi dS) space, an unbounded number of massless modes also pop up in the limit $\Lambda \to 0$.

As demonstrated in [12], applying the AdS-DC to dS space could help elucidate the radiative stability of the cosmological hierarchy $\Lambda/M_p^4 \sim 10^{-120}$, because it connects the size of the compact space $R_\perp$ to the dark energy scale $\Lambda^{-1/4}$ via $R_\perp \sim \lambda \Lambda^{-1/4}$, where the proportionality factor is estimated to be within the range $10^{-1} < \lambda < 10^{-4}$. Actually, the previous relation between $R_\perp$ and Λ derives from constraints by theory and experiment. On the one hand, since the associated Kaluza-Klein (KK) tower contains massive spin-2 bosons, the Higuchi bound [13] provides an absolute upper limit to α, whereas explicit string calculations of the vacuum energy (see, e.g., [14, 15, 16, 17]) yield a lower bound on α. All in all, the theoretical constraints lead to $1/4 \leq \alpha \leq 1/2$. On the other hand, experimental arguments (e.g., constraints on deviations from Newton's gravitational inverse-square law [18] and neutron star heating [19]) lead to the conclusion encapsulated in $R_\perp \sim \lambda \Lambda^{-1/4}$; namely, that there is one extra dimension of radius $R_\perp$ in the micron range, and that the lower bound for $\alpha = 1/4$ is basically saturated [12]. A theoretical amendment on the connection between the cosmological and KK mass scales confirms $\alpha = 1/4$ [20]. Assembling all this together, we can conclude that the KK tower of the new (dark) dimension opens up at the mass scale $m_{\rm KK} \sim 1/R_\perp$. Within this set-up, the 5-dimensional Planck scale (or species scale where gravity becomes strong [21, 22, 23, 24]) is given by $M_* \sim m_{\rm KK}^{1/3} M_p^{2/3}$.

It is of course interesting to explore whether there is a relation between the supersymmetry (SUSY) breaking scale and the measured value of the dark energy density Λ. Such a relation can be derived by combining two quantum gravity consistency swampland constraints, which tie Λ and the gravitino mass $M_{3/2}$, to the mass scale of a light KK tower and, therefore, to the UV cut-off of the EFT [25, 26, 27]. One can then use the constraint on $M_{3/2}$ to infer the implications of the dark dimension scenario for the scale of supersymmetry breaking. In general, one can distinguish two situations. In the first case, the gravitino mass and the cosmological constant are related to the same tower of states. This is arguably the simplest scenario, in which the natural scale for SUSY signatures is of order $\Lambda^{1/8} \sim$ TeV, and therefore is within reach of LHC and of the next generation of hadron colliders [28]. In the second case, $M_{3/2}$ and Λ are related to different towers. This scenario requires a decoupling of the gravitino mass from the cosmological constant and is thus more difficult to realize in concrete models.

Possible string theory and effective supergravity realizations of the dark dimension scenario with broken supersymmetry are discussed in [28].

3. DARK MATTER CANDIDATES

After the big bang, the cosmological energy density scales with time t as $\rho \sim 1/(Gt^2)$ and the density needed for a region of mass $M_{\rm BH}$ to collapse within its Schwarzschild radius is $\rho \sim c^6/(G^3 M_{\rm BH}^2)$, that being so primordial black holes (PBHs) would initially have around the cosmological horizon mass [29]

$$M_{\rm BH} \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23}\,{\rm s}}\right) {\rm g}. \tag{2}$$

This means that a black hole would have the Planck mass ($M_p \sim 10^{-5}$ g) if they formed at the Planck time (10^{-43} s), $1\,M_\odot$ if they formed at the QCD epoch (10^{-5} s), and $10^5\,M_\odot$ if they formed at $t \sim 1$ s, comparable to the mass of the holes thought to reside in galactic nuclei. This back-of-the-envelope calculation suggests that PBHs could span an enormous mass range. Despite the fact that the mass spectrum of these PBHs is not set in stone, on cosmological scales they would behave like a typical CDM particle. However, an all-dark-matter interpretation in terms of PBHs is severely constrained by observations [29, 30, 31]. The extragalactic γ-ray background [32] and on the CMB spectrum [33] constrain PBH evaporation of black holes with masses $\lesssim 10^{17}$ g, whereas the non-observation of microlensing events from the MACHO [34], EROS [35], Kepler [36], Icarus [37], OGLE [38] and Subaru-HSC [39] collaborations constrain black holes with masses $\gtrsim 10^{21}$ g. Of course it is of interest to see whether new effects associated to the dark dimension could relax these bounds.

It has long been known that microscopic black holes—with Schwarzschild radii smaller than the size of the dark dimension—are quite different: they are bigger, colder, and longer-lived than a usual four-dimensional (4D) black hole of the same mass [40]. Indeed, black holes radiate all particle species lighter than or comparable to their temperature, which in four dimensions is related to the mass of the black hole by

$$T_{\rm BH} = \frac{M_p^2}{8\pi M_{\rm BH}} \sim \left(\frac{M_{\rm BH}}{10^{16}\,{\rm g}}\right)^{-1} {\rm MeV}, \tag{3}$$

whereas for five dimensional black holes the temperature mass relation is found to be [41]

$$T_{\rm BH} = \sqrt{\frac{3}{64}}\frac{1}{\pi}\frac{M_p \Lambda^{1/8}}{\lambda^{1/2} M_{\rm BH}^{1/2}} \sim \left(\frac{M_{\rm BH}}{10^{10}\,{\rm g}}\right)^{-1/2} {\rm MeV}, \tag{4}$$

where we have taken $\lambda \sim 10^{-3}$ as suggested by astrophysical observations [42, 43]. It is evident that 5D black holes are colder than 4D black holes of the same mass. The Hawking radiation causes a 4D black hole to lose mass at the following rate [44]

$$\begin{aligned} \left.\frac{dM_{\rm BH}}{dt}\right|_{\rm evap} &= -\frac{M_p^2}{30720\pi M_{\rm BH}^2}\sum_i c_i\left(T_{\rm BH}\right)\tilde{f}\Gamma_s \\ &\sim -7.5\times 10^{-8}\left(\frac{M_{\rm BH}}{10^{16}\,{\rm g}}\right)^{-2}\sum_i c_i\left(T_{\rm BH}\right)\tilde{f}\Gamma_s\,{\rm g/s}, \end{aligned} \tag{5}$$

whereas a 5D black hole has an evaporation rate of [41]

$$\begin{aligned} \left.\frac{dM_{\rm BH}}{dt}\right|_{\rm evap} &= -\frac{\Lambda^{1/4} M_{\rm Pl}^2}{640\pi\lambda M_{\rm BH}}\sum_i c_i\left(T_{\rm BH}\right)\tilde{f}\Gamma_s \\ &\sim -1.3\times 10^{-12}\left(\frac{M_{\rm BH}}{10^{16}\,{\rm g}}\right)^{-1}\sum_i c_i\left(T_{\rm BH}\right)\tilde{f}\Gamma_s\,{\rm g/s}, \end{aligned} \tag{6}$$

where $c_i(T_{\rm BH})$ counts the number of internal degrees of freedom of particle species i of mass m_i satisfying $m_i \ll T_{\rm BH}$, $\tilde{f} = 1$ ($\tilde{f} = 7/8$) for bosons (fermions), and where $\Gamma_{s=1/2} \approx 2/3$ and $\Gamma_{s=1} \approx 1/4$ are the (spin-weighted) dimensionless greybody factors normalized to the black hole surface area [45]. In the spirit of [46], graviton emission can be neglected because the KK modes are excitations in the full transverse space, and so their overlap with the small (higher-dimensional) black holes is suppressed by the geometric factor $(r_s/R_\perp)^2$ relative to the brane fields, where r_s is the Schwarzschild radius [47]. Thus, the geometric suppression precisely compensates for the enormous number of modes, and the total contribution of all KK modes is only the same order as that from a single brane field.

Now, integrating (5) and (6) it is easily seen that 5D black holes live longer than 4D black holes of the same mass. Armed with this result a straightforward calculation shows that for a species scale of $\mathcal{O}(10^9\,{\rm GeV})$, an all-dark-matter interpretation in terms of 5D black holes must be feasible for masses in the range $10^{14} < M_{\rm BH}/{\rm g} < 10^{21}$ [41]. This range is extended compared to that in the 4D theory by 3 orders of magnitude in the low mass region.

An astonishing coincidence is that the size of the dark dimension $R_\perp \sim$ wavelength of visible light. This means that the Schwarzschild radius of 5D black holes is well below the wavelength of light. For point-like lenses, this is the critical length where geometric optics breaks down and the effects of wave optics suppress the magnification, obstructing the sensitivity to 5D PBH microlensing signals [39].

It was observed in [48] that the universal coupling of the SM fields to the massive spin-2 KK excitations of the graviton in the dark dimension provides an alternative dark matter candidate. Within this model the cosmic evolution of the hidden sector is primarily dominated by "dark-to-dark" decays, yielding a specific realization of the dynamical dark matter framework [49]. Consider a tower of equally spaced dark gravitons, indexed by an integer l, and with mass $m_l = l m_{\rm KK}$. The partial decay width of KK graviton l to SM fields is found to be,

$$\Gamma^l_{\rm SM} = \frac{\tilde{\lambda}^2 m_{\rm KK}^3 l^3}{80\pi M_{\rm Pl}^2}, \tag{7}$$

where $\tilde{\lambda}$ takes into account all the available decay channels and is a function of time [50].

In the absence of isometries in the dark dimension, which is the common expectation, the KK momentum of the dark tower is not conserved. This means that a dark graviton of KK quantum n can decay to two other ones, with quantum numbers n_1 and n_2. If the KK quantum violation can go up to δn, the number of available channels is roughly $l\delta n$. In addition, because the decay is almost at threshold, the phase space factor is roughly the velocity of decay products, $v_{\rm r.m.s.} \sim \sqrt{m_{\rm KK}\delta n/m_l}$. Putting all this together we obtain the total decay width,

$$\begin{aligned} \Gamma^l_{\rm tot} &\sim \sum_{l'<l}\ \sum_{0<l''<l-l'} \Gamma^l_{l'l''} \sim \beta^2 \frac{m_l^3}{M_{\rm Pl}^2} \times \frac{m_l}{m_{\rm KK}}\delta n \times \sqrt{\frac{m_{\rm KK}\delta_n}{m_l}} \\ &\sim \beta^2 \delta n^{3/2} \frac{m_l^{7/2}}{M_{\rm Pl}^2 m_{\rm KK}^{1/2}}, \end{aligned} \tag{8}$$

where β parametrizes our ignorance of decays in the dark dimension [48].

To estimate the time evolution of the dark matter mass assume that for times larger than $1/\Gamma^l_{\rm tot}$ dark matter which is heavier than the corresponding m_l has already decayed, and so it follows that

$$m_l \sim \left(\frac{M_{\rm Pl}^4 m_{\rm KK}}{\beta^4 \delta n^3}\right)^{1/7} t^{-2/7}, \tag{9}$$

where t indicates the time elapsed since the big bang [48].

Consistency with CMB anisotropies requires $\Gamma^{l}_{\gamma\gamma} < 5 \times 10^{-25}\,\mathrm{s}^{-1}$ between the last scattering surface and reionization [51]. Taking $\tilde{\lambda} = 1$ (to set out the decay into photons) and using (7) it follows that the CMB requirement is satisfied for $l \lesssim 10^8$ at the time $t_{\rm MR} \sim 6 \times 10^4$ yr of matter-radiation equality. In other words, by setting $\tilde{\lambda} \sim 1$ and $m_l(t_{\rm MR}) \lesssim 1$ MeV, the evolution of m_l with cosmic time given in (9) is such that at the last scattering surface the dominant KK state in the dynamical dark matter ensemble has the correct decay width to accommodate the CMB constraints [52].

Now, we have seen that dark matter decay gives the daughter particles a velocity kick. Self-gravitating dark-matter halos that have a virial velocity smaller than this velocity kick may be disrupted by these particle decays. Consistency with existing data requires roughly $\delta n \sim 1$, and $\beta \sim 635$ [53]. For selected fiducial parameters, the cosmic evolution of the incredible bulk predicts via (9) a dominant particle mass of ~900 keV at CMB, of ~500 keV in the Dark Ages, of ~150 keV at Cosmic Dawn, and of ~50 keV in the local universe. This is in sharp contrast to typical dark matter decay scenarios with one unstable particle (such as sterile neutrinos [54]). Simultaneous observations of signals at Cosmic Dawn and in the local universe could constitute the smoking gun of the incredible bulk [55].

For many purposes, a black hole can be replaced by a bound state of gravitons [56]. As a matter of fact, a correspondence between 5D PBHs and massive KK gravitons as dark matter candidates has been conjectured in [57].

The radion stabilizing the dark dimension could be yet another dark matter contender [58]. This is because in principle the radion could be ultralight, and if this were the case it would serve as a fuzzy dark matter candidate. A simple cosmological production mechanism brings into play unstable KK graviton towers which are fueled by the decay of the inflaton. As in the previous model, the cosmic evolution of the dark sector is mostly driven by "dark-to-dark" decay processes that regulate the decay of KK gravitons within the dark tower, conveying another realization of the dynamical dark matter framework [49]. In the spirit of [59], within this model it is assumed that the intra-KK decays in the bulk carry a spontaneous breakdown of the translational invariance in the compact space, such that the 5D momenta are not conserved (but now $\delta n \gg 1$). Armed with these two reasonable assumptions it is straightforward to see that the energy the inflaton deposited in the KK tower should have collapsed all into the radion well before BBN.

4. NEUTRINO MASSES AND MIXING

The dark dimension scenario provides a profitable arena to realize an old idea for explaining the smallness of neutrino masses by introducing the right-handed neutrinos as 5D bulk states with Yukawa couplings to the left-handed lepton and Higgs doublets that are localized states on the SM brane stack [60, 61, 62]. The neutrino masses are then suppressed due to the wave function of the bulk states.

More indicatively, the generation of neutrino masses originates in 5D bulk-brane interactions of the form

$$\mathscr{L} \supset h_{ij} \overline{L}_i \tilde{H} \Psi_j (y = 0), \tag{10}$$

where $\tilde{H} = -i\sigma_2 H^*$, L_i denotes the lepton doublets (localized on the SM brane), Ψ_j stands for the 3 bulk (right-handed) R-neutrinos evaluated at the position of the SM brane, $y = 0$ in the fifth-dimension coordinate y, and h_{ij} are coupling constants. This gives a coupling with the L-neutrinos of the form $\langle H \rangle \overline{\nu}_{L_i} \Psi_j (y = 0)$, where $\langle H \rangle = 175$ GeV is the Higgs vacuum expectation value. Expanding Ψ_j into modes canonically normalized leads for each of them to a Yukawa 3×3 matrix suppressed by the square root of the volume of the bulk $\sqrt{\pi R_\perp M_s}$, i.e.,

$$Y_{ij} = \frac{h_{ij}}{\sqrt{\pi R_\perp M_s}} \sim h_{ij} \frac{M_s}{M_p}, \tag{11}$$

where $M_s \lesssim M_*$ is the string scale, and where in the second rendition we have dropped factors of π's and of the string coupling.

Now, neutrino oscillation data can be well-fitted in terms of two nonzero differences $\Delta m^2_{ij} = m^2_i - m^2_j$ between the squares of the masses of the three mass eigenstates; namely, $\Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5}\,\mathrm{eV}^2$ and $\Delta m^2_{32} = (2.453 \pm 0.033) \times 10^{-3}\,\mathrm{eV}^2$ or $\Delta m^2_{32} = -(2.536 \pm 0.034) \times 10^{-3}\,\mathrm{eV}^2$ [1]. It is easily seen that to obtain the correct order of magnitude of neutrino masses the coupling h_{ij} should be of order 10^{-4} to 10^{-5} for $10^9 \lesssim M_s/\mathrm{GeV} \lesssim 10^{10}$.

Note that KK modes of the 5D R-neutrino fields behave as an infinite tower of sterile neutrinos, with masses proportional to $m_{\rm KK}$. However, only the lower mass states of the tower mix with the active SM neutrinos in a pertinent fashion. The non-observation of neutrino disappearance from oscillations into sterile neutrinos at long- and short-baseline experiments places a 90% CL upper limit $R_\perp < 0.4\,\mu$m for the normal neutrino ordering, and $R_\perp < 0.2\,\mu$m for the inverted neutrino ordering [63, 64].[1] This set of parameters corresponds to $\lambda \lesssim 10^{-3}$ and so $m_{\rm KK} \gtrsim 2.5$ eV [55].

Before proceeding, it is important to stress that the upper bounds on $R_\perp$ discussed in the previous paragraph are sensitive to assumptions of the 5$^{\rm th}$ dimension geometry. Moreover, in the presence of bulk masses [65, 66], the mixing of the first KK modes to active neutrinos can be suppressed, and therefore the aforementioned bounds on $R_\perp$ can be avoided [67, 68]. It is also worth mentioning that such bulk masses have the potential to increase the relative importance of the higher KK modes, yielding distinct oscillation signatures via neutrino disappearance/appearance effects.

[1] We arrived at these upper bounds by looking at the low mass limit of the lightest neutrino state in Figure 6 of [64] and rounding the numbers to one significant figure.

Non-minimal extensions of the dark dimension, in which $M_{3/2}$ and Λ have different KK towers, allow a high-scale SUSY breaking and can therefore host a rather heavy gravitino together with a modulino with a mass of about 50 eV [69]. For a particular example, we note that the modulino could be the fermionic partner of the radion.[2] These models with high-scale SUSY breaking are fully predictive through neutrino-modulino oscillations [70] which can be confronted with data to be collected by experiments at the Forward Physics Facility [71, 72].

A seemingly different, but in fact closely related subject is the the *sharpened* version of the weak gravity conjecture forbidding the presence of non-SUSY AdS vacua supported by fluxes in a consistent quantum gravity theory [73]. This is because (unless the gravitino is very light, with mass in the meV range) *neutrinos have to be Dirac with right-handed states propagating in the bulk so that the KK neutrino towers compensate for the graviton tower to maintain stable dS vacua* [68].

5. MESOSCOPIC EXTRA DIMENSION FROM 5D INFLATION

It is unnatural to entertain that the size of the dark dimension would remain fixed during the evolution of the Universe right at the species scale. One possible mechanism to accommodate this hierarchy is to inflate the size of the dark dimension. The required inflationary phase can be described by a 5D dS (or approximate) solution of Einstein equations, with cosmological constant and a 5D Planck scale $M_* \sim 10^9$ GeV [55]. All dimensions (compact and non-compact) expand exponentially in terms of the 5D proper time. It is straightforward to see that this set-up requires about 42 e-folds to expand the 5th dimension from the fundamental length $\mathcal{O}(M_*^{-1})$ to the micron size $\mathcal{O}(R_\perp)$. At the end of 5D inflation, or at any given moment, one can interpret the solution in terms of 4D fields using 4D Planck units from the relation $M_p^2 = M_*^3 R$, which amounts going to the 4D Einstein frame. This implies that if $M_*^{-1} \leq R \leq R_\perp$ expands N e-folds, then the 3D space would expand $3N/2$ e-folds as a result of a uniform 5D inflation. Altogether, the 3D space has expanded by about 60 e-folds to solve the horizon problem, while connecting this particular solution to the generation of large size extra dimension.

Besides solving the horizon problem, 4D slow-roll inflation predicts an approximate scale-invariant Harrison-Zel'dovich power spectrum of primordial density perturbations [74, 75] consistent with CMB observations [76]. This is due to the fact that the 2-point function of a massless minimally coupled scalar field in dS space behaves logarithmically at distances larger than the cosmological horizon, a property which is though valid for any spacetime dimensionality [77]. When some dimensions are however compact, this behaviour is expected to hold for distances smaller than the compactification length, while deviating from scale invariance at larger distances, potentially conflicting with observations at large angles. Remarkably, consistency 5D inflation with CMB observations is maintained if the size of the dark dimension is larger than about a micron, implying a change of behaviour in the power spectrum at angles larger than 10 degrees, corresponding to multiple moments $l \lesssim 30$, where experimental errors are getting large [3]. Actually, the scale invariance of the power spectrum is obtained upon summation over the contribution of the inflaton KK-modes' fluctuations that correspond to a tower of scalars from the 4D point of view. The tensor perturbations have been computed in [78]. The tensor-to-scalar ratio is found to be $r = 24\epsilon_V$, and so the 95% CL upper limit $r < 0.032$ (derived using a combination of BICEP/Keck 2018 and *Planck* data) [79, 80] places an experimental constraint on the potential slow-roll parameter: $\epsilon_V < 0.0013$.

Another interesting feature of 5D inflation is that the radion can be stabilized in a local (metastable) dS vacuum, using the contributions of bulk field gradients [81] or of the Casimir energy, assuming a mass for the bulk R-handed neutrinos of the same order of magnitude [82].

6. TENSIONS IN COSMOLOGY

Over the last few years, low- and high-redshift observations set off tensions in the measurement of the present-day expansion rate H_0 and in the determination of the amplitude of the matter clustering in the late Universe (parameterized by S_8) [83]. It was recently noted that both these tensions can be resolved if the cosmological constant parametrizing the dark energy content switches its sign at a critical redshift $z_c \sim 2$ [84]. In addition, the so-called Λ_sCDM model can accommodate the BAO Lyman-α disicrepancy [85] and is in agreement with the otherwise puzzling JWST observations [86]. However, the AdS distance conjecture suggests that the postulated switch in sign of the cosmological constant at zero temperature seems unlikely because the AdS vacua are an infinite distance appart from dS vacua in moduli space [11]. A possible explanation for the required AdS $\to$ dS crossover transition in the vacuum energy can be obtained using the Casimir forces of fields inhabiting the dark dimension [87]. Using entropy arguments it is easily seen that any AdS $\to$ dS transition between metastable vacua must be accompanied by a reduction of the species scale where gravity becomes strong.

7. CONCLUDING REMARKS

We have seen that the dark dimension scenario carries with it a rich phenomenology:

(i) It provides a profitable arena to accommodate a very light gravitino.

[2] In the standard moduli stabilization by fluxes, all complex structure moduli and the dilaton are stabilized in a supersymmetric way while Kähler class moduli need an input from SUSY breaking. The radion is Kähler class and exists in a model independent fashion within the dark dimension scenario.

(ii) It encompasses a framework for primordial black holes, KK gravitons, and a fuzzy radion to emerge as interesting dark matter candidates.

(iii) It also encompasses an interesting framework for studying cosmology and astroparticle physics.

(iv) It provides a natural set up for R-neutrinos propagating in the bulk to accommodate neutrino masses in the range $10^{-4} < m_\nu/\text{eV} < 10^{-1}$, despite the lack of any fundamental scale higher than M_*. The suppressed neutrino masses are not the result of a see-saw mechanism, but rather because the bulk modes have couplings suppressed by the volume of the dark dimension (akin to the weakness of gravity at long distances).

We have also seen that uniform 5D inflation can relate the causal size of the observable universe to the present weakness of gravitational interactions by blowing up an extra compact dimension from the microscopic fundamental length of gravity to a large size in the micron range, as required by the dark dimension scenario. Moreover, uniform 5D inflation can lead to an approximate scale invariant power spectrum of primordial density perturbations consistent with observations of CMB anisotropies. The tensor-to-scalar ratio is also consistent with observations. A rough estimate of the magnitude of isocurvature perturbations based on entropy perturbations indicates that they are suppressed. A dedicated investigation along these lines is obviously important to be done.

ACKNOWLEDGMENTS

The work of Luis A. Anchordoqui is supported by the U.S. National Science Foundation (NSF Grant PHY-2112527). The work of Dieter Lüst is supported by the Origins Excellence Cluster and by the German-Israel-Project (DIP) on Holography and the Swampland.

References

[1] R. L. Workman et al. [Particle Data Group], *Review of Particle Physics*, PTEP **2022**, 083C01 (2022) doi:10.1093/ptep/ptac097
[2] A. G. Riess, W. Yuan, L. M. Macri, D. Scolnic, D. Brout, S. Casertano, D. O. Jones, Y. Murakami, L. Breuval, and T. G. Brink, et al. *A comprehensive measurement of the local value of the Hubble constant with 1 km s^{-1} Mpc^{-1} uncertainty from the Hubble Space Telescope and the SH0ES Team*, Astrophys. J. Lett. **934** (2022) no.1, L7 doi:10.3847/2041-8213/ac5c5b [arXiv:2112.04510 [astro-ph.CO]].
[3] L. A. Anchordoqui and I. Antoniadis, *Large extra dimensions from higher-dimensional inflation*, [arXiv:2310.20282 [hep-ph]].
[4] N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *The Hierarchy problem and new dimensions at a millimeter*, Phys. Lett. B **429** (1998), 263–272 doi:10.1016/S0370-2693(98)00466-3 [arXiv:hep-ph/9803315 [hep-ph]].
[5] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *New dimensions at a millimeter to a Fermi and superstrings at a TeV*, Phys. Lett. B **436** (1998), 257–263 doi:10.1016/S0370-2693(98)00860-0 [arXiv:hep-ph/9804398 [hep-ph]].
[6] C. Vafa, *The string landscape and the swampland*, [arXiv:hep-th/0509212 [hep-th]].
[7] M. van Beest, J. Calderón-Infante, D. Mirfendereski, and I. Valenzuela, *Lectures on the Swampland Program in string compactifications*, Phys. Rept. **989**, 1–50 (2022) doi:10.1016/j.physrep.2022.09.002 [arXiv:2102.01111 [hep-th]].
[8] E. Palti, *The swampland: introduction and review*, Fortsch. Phys. **67**, no.6, 1900037 (2019) doi:10.1002/prop.201900037 [arXiv:1903.06239 [hep-th]].
[9] N. B. Agmon, A. Bedroya, M. J. Kang, and C. Vafa, *Lectures on the string landscape and the swampland*, [arXiv:2212.06187 [hep-th]].
[10] H. Ooguri and C. Vafa, *On the Geometry of the String Landscape and the Swampland*, Nucl. Phys. B **766**, 21–33 (2007) doi:10.1016/j.nuclphysb.2006.10.033 [arXiv:hep-th/0605264 [hep-th]].
[11] D. Lüst, E. Palti, and C. Vafa, *AdS and the Swampland*, Phys. Lett. B **797**, 134867 (2019) doi:10.1016/j.physletb.2019.134867 [arXiv:1906.05225 [hep-th]].
[12] M. Montero, C. Vafa, and I. Valenzuela, *The dark dimension and the Swampland*, JHEP **02**, 022 (2023) doi:10.1007/JHEP02(2023)022 [arXiv:2205.12293 [hep-th]].
[13] A. Higuchi, *Forbidden mass range for spin-2 field theory in de Sitter space-time*, Nucl. Phys. B **282** (1987), 397–436 doi:10.1016/0550-3213(87)90691-2
[14] H. Itoyama and T. R. Taylor, *Supersymmetry restoration in the compactified $O(16) \times O(16)$-prime heterotic string theory*, Phys. Lett. B **186** (1987), 129–133 doi:10.1016/0370-2693(87)90267-X
[15] H. Itoyama and T. R. Taylor, *Small cosmological constant in string models*, FERMILAB-CONF-87-129-T.
[16] I. Antoniadis and C. Kounnas, *Superstring phase transition at high temperature*, Phys. Lett. B **261** (1991), 369–378 doi:10.1016/0370-2693(91)90442-S
[17] Q. Bonnefoy, E. Dudas, and S. Lüst, *On the weak gravity conjecture in string theory with broken supersymmetry*, Nucl. Phys. B **947** (2019), 114738 doi:10.1016/j.nuclphysb.2019.114738 [arXiv:1811.11199 [hep-th]].
[18] J. G. Lee, E. G. Adelberger, T. S. Cook, S. M. Fleischer, and B. R. Heckel, *New test of the gravitational $1/r^2$ law at separations down to 52 μm*, Phys. Rev. Lett. **124**, no.10, 101101 (2020) doi:10.1103/PhysRevLett.124.101101 [arXiv:2002.11761 [hep-ex]].
[19] S. Hannestad and G. G. Raffelt, *Supernova and neutron star limits on large extra dimensions reexamined*, Phys. Rev. D **67**, 125008 (2003) [erratum: Phys. Rev. D **69**, 029901 (2004)] doi:10.1103/PhysRevD.69.029901 [arXiv:hep-ph/0304029 [hep-ph]].
[20] L. A. Anchordoqui, I. Antoniadis, D. Lüst, and S. Lüst, *On the cosmological constant, the KK mass scale, and the cut-off dependence in the dark dimension scenario*, Eur. Phys. J. C **83**, no.11, 1016 (2023) doi:10.1140/epjc/s10052-023-12206-2 [arXiv:2309.09330 [hep-th]].
[21] G. Dvali, *Black holes and large N species solution to the hierarchy problem*, Fortsch. Phys. **58**, 528–536 (2010) doi:10.1002/prop.201000009 [arXiv:0706.2050 [hep-th]].
[22] G. Dvali and M. Redi, *Black hole bound on the number of species and quantum gravity at LHC*, Phys. Rev. D **77**, 045027 (2008) doi:10.1103/PhysRevD.77.045027 [arXiv:0710.4344 [hep-th]].
[23] N. Cribiori, D. Lüst, and G. Staudt, *Black hole entropy and moduli-dependent species scale*, Phys. Lett. B **844**, 138113 (2023) doi:10.1016/j.physletb.2023.138113 [arXiv:2212.10286 [hep-th]].
[24] D. van de Heisteeg, C. Vafa, M. Wiesner, and D. H. Wu, *Species scale in diverse dimensions*, [arXiv:2310.07213 [hep-th]].

[25] I. Antoniadis, C. Bachas, D. C. Lewellen, and T. N. Tomaras, *On supersymmetry breaking in superstrings*, Phys. Lett. B **207**, 441–446 (1988) doi:10.1016/0370-2693(88)90679-X

[26] N. Cribiori, D. Lüst, and M. Scalisi, *The gravitino and the swampland*, JHEP **06**, 071 (2021) doi:10.1007/JHEP06(2021)071 [arXiv:2104.08288 [hep-th]].

[27] A. Castellano, A. Font, A. Herraez, and L. E. Ibáñez, *A gravitino distance conjecture*, JHEP **08**, 092 (2021) doi:10.1007/JHEP08(2021)092 [arXiv:2104.10181 [hep-th]].

[28] L. A. Anchordoqui, I. Antoniadis, N. Cribiori, D. Lüst, and M. Scalisi, *The Scale of Supersymmetry Breaking and the Dark Dimension*, JHEP **05**, 060 (2023) doi:10.1007/JHEP05(2023)060 [arXiv:2301.07719 [hep-th]].

[29] B. Carr and F. Kuhnel, *Primordial black holes as dark matter: Recent developments*, Ann. Rev. Nucl. Part. Sci. **70** (2020), 355–394 doi:10.1146/annurev-nucl-050520-125911 [arXiv:2006.02838 [astro-ph.CO]].

[30] A. M. Green and B. J. Kavanagh, *Primordial Black Holes as a dark matter candidate*, J. Phys. G **48** (2021) no.4, 043001 doi:10.1088/1361-6471/abc534 [arXiv:2007.10722 [astro-ph.CO]].

[31] P. Villanueva-Domingo, O. Mena, and S. Palomares-Ruiz, *A brief review on primordial black holes as dark matter*, Front. Astron. Space Sci. **8**, 87 (2021) doi:10.3389/fspas.2021.681084 [arXiv:2103.12087 [astro-ph.CO]].

[32] B. J. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, *New cosmological constraints on primordial black holes*, Phys. Rev. D **81**, 104019 (2010) doi:10.1103/PhysRevD.81.104019 [arXiv:0912.5297 [astro-ph.CO]].

[33] S. Clark, B. Dutta, Y. Gao, L. E. Strigari, and S. Watson, *Planck constraint on relic primordial black holes*, Phys. Rev. D **95**, no.8, 083006 (2017) doi:10.1103/PhysRevD.95.083006 [arXiv:1612.07738 [astro-ph.CO]].

[34] R. A. Allsman et al. [MACHO], *MACHO project limits on black hole dark matter in the 1–30 solar mass range*, Astrophys. J. Lett. **550**, L169 (2001) doi:10.1086/319636 [arXiv:astro-ph/0011506 [astro-ph]].

[35] P. Tisserand et al. [EROS-2], *Limits on the MACHO content of the Galactic halo from the EROS-2 survey of the Magellanic clouds*, Astron. Astrophys. **469**, 387–404 (2007) doi:10.1051/0004-6361:20066017 [arXiv:astro-ph/0607207 [astro-ph]].

[36] K. Griest, A. M. Cieplak, and M. J. Lehner, *Experimental limits on primordial black hole dark matter from the first 2 yr of Kepler data*, Astrophys. J. **786**, no.2, 158 (2014) doi:10.1088/0004-637X/786/2/158 [arXiv:1307.5798 [astro-ph.CO]].

[37] M. Oguri, J. M. Diego, N. Kaiser, P. L. Kelly, and T. Broadhurst, *Understanding caustic crossings in giant arcs: characteristic scales, event rates, and constraints on compact dark matter*, Phys. Rev. D **97**, no.2, 023518 (2018) doi:10.1103/PhysRevD.97.023518 [arXiv:1710.00148 [astro-ph.CO]].

[38] H. Niikura, M. Takada, S. Yokoyama, T. Sumi, and S. Masaki, *Constraints on Earth-mass primordial black holes from OGLE 5-year microlensing events*, Phys. Rev. D **99**, no.8, 083503 (2019) doi:10.1103/PhysRevD.99.083503 [arXiv:1901.07120 [astro-ph.CO]].

[39] D. Croon, D. McKeen, N. Raj, and Z. Wang, *Subaru-HSC through a different lens: Microlensing by extended dark matter structures*, Phys. Rev. D **102**, no.8, 083021 (2020) doi:10.1103/PhysRevD.102.083021 [arXiv:2007.12697 [astro-ph.CO]].

[40] P. C. Argyres, S. Dimopoulos, and J. March-Russell, Phys. Lett. B **441** (1998), 96–104 doi:10.1016/S0370-2693(98)01184-8 [arXiv:hep-th/9808138 [hep-th]].

[41] L. A. Anchordoqui, I. Antoniadis, and D. Lüst, *Dark dimension, the swampland, and the dark matter fraction composed of primordial black holes*, Phys. Rev. D **106**, no.8, 086001 (2022) doi:10.1103/PhysRevD.106.086001 [arXiv:2206.07071 [hep-th]].

[42] L. A. Anchordoqui, *Dark dimension, the swampland, and the origin of cosmic rays beyond the Greisen-Zatsepin-Kuzmin barrier*, Phys. Rev. D **106** (2022) no.11, 116022 doi:10.1103/PhysRevD.106.116022 [arXiv:2205.13931 [hep-ph]].

[43] N. T. Noble, J. F. Soriano, and L. A. Anchordoqui, *Probing the Dark Dimension with Auger data*, Phys. Dark Univ. **42** (2023), 101278 doi:10.1016/j.dark.2023.101278 [arXiv:2306.03666 [hep-ph]].

[44] C. Keith and D. Hooper, *511 keV excess and primordial black holes*, Phys. Rev. D **104** (2021) no.6, 063033 doi:10.1103/PhysRevD.104.063033 [arXiv:2103.08611 [astro-ph.CO]].

[45] L. Anchordoqui and H. Goldberg, *Black hole chromosphere at the CERN LHC*, Phys. Rev. D **67** (2003), 064010 doi:10.1103/PhysRevD.67.064010 [arXiv:hep-ph/0209337 [hep-ph]].

[46] R. Emparan, G. T. Horowitz, and R. C. Myers, *Black holes radiate mainly on the brane*, Phys. Rev. Lett. **85**, 499–502 (2000) doi:10.1103/PhysRevLett.85.499 [arXiv:hep-th/0003118 [hep-th]].

[47] R. C. Myers and M. J. Perry, *Black holes in higher dimensional space-times*, Annals Phys. **172**, 304 (1986) doi:10.1016/0003-4916(86)90186-7

[48] E. Gonzalo, M. Montero, G. Obied, and C. Vafa, *Dark Dimension Gravitons as Dark Matter*, JHEP **11**, 109 (2023) doi:10.1007/JHEP11(2023)109 [arXiv:2209.09249 [hep-ph]].

[49] K. R. Dienes and B. Thomas, *Dynamical dark matter I: Theoretical overview*, Phys. Rev. D **85**, 083523 (2012) doi:10.1103/PhysRevD.85.083523 [arXiv:1106.4546 [hep-ph]].

[50] L. J. Hall and D. Tucker-Smith, *Cosmological constraints on theories with large extra dimensions*, Phys. Rev. D **60**, 085008 (1999) doi:10.1103/PhysRevD.60.085008 [arXiv:hep-ph/9904267 [hep-ph]].

[51] T. R. Slatyer and C. L. Wu, *General Constraints on Dark Matter Decay from the Cosmic Microwave Background*, Phys. Rev. D **95**, no.2, 023010 (2017) doi:10.1103/PhysRevD.95.023010 [arXiv:1610.06933 [astro-ph.CO]].

[52] J. A. P. Law-Smith, G. Obied, A. Prabhu, and C. Vafa, *Astrophysical Constraints on Decaying Dark Gravitons*, [arXiv:2307.11048 [hep-ph]].

[53] G. Obied, C. Dvorkin, E. Gonzalo, and C. Vafa, *Dark Dimension and Decaying Dark Matter Gravitons*, [arXiv:2311.05318 [astro-ph.CO]].

[54] K. N. Abazajian, *Sterile neutrinos in cosmology*, Phys. Rept. **711-712**, 1–28 (2017) doi:10.1016/j.physrep.2017.10.003 [arXiv:1705.01837 [hep-ph]].

[55] L. A. Anchordoqui, I. Antoniadis, and D. Lüst, *Aspects of the dark dimension in cosmology*, Phys. Rev. D **107**, no.8, 083530 (2023) doi:10.1103/PhysRevD.107.083530 [arXiv:2212.08527 [hep-ph]].

[56] G. Dvali and C. Gomez, *Black hole's quantum N-portrait*, Fortsch. Phys. **61**, 742–767 (2013) doi:10.1002/prop.201300001 [arXiv:1112.3359 [hep-th]].

[57] L. A. Anchordoqui, I. Antoniadis, and D. Lüst, *The dark universe: Primordial black hole $\leftrightharpoons$ dark graviton gas connection*, Phys. Lett. B **840**, 137844 (2023) doi:10.1016/j.physletb.2023.137844 [arXiv:2210.02475 [hep-th]].

[58] L. A. Anchordoqui, I. Antoniadis, and D. Lüst, *Fuzzy Dark Matter, the dark dimension, and the pulsar timing array signal*, [arXiv:2307.01100 [hep-ph]].

[59] R. N. Mohapatra, S. Nussinov, and A. Perez-Lorenzana, *Large extra dimensions and decaying KK recurrences*, Phys. Rev. D **68**, 116001 (2003) doi:10.1103/PhysRevD.68.116001 [arXiv:hep-ph/0308051 [hep-ph]].

[60] K. R. Dienes, E. Dudas, and T. Gherghetta, *Neutrino oscillations without neutrino masses or heavy mass scales: A Higher dimensional seesaw mechanism*, Nucl. Phys. B **557**, 25 (1999) doi:10.1016/S0550-3213(99)00377-6 [arXiv:hep-ph/9811428 [hep-ph]].

[61] N. Arkani-Hamed, S. Dimopoulos, G. R. Dvali, and J. March-Russell, *Neutrino masses from large extra dimensions*, Phys. Rev. D **65**, 024032 (2001) doi:10.1103/PhysRevD.65.024032 [arXiv:hep-ph/9811448 [hep-ph]].

[62] G. R. Dvali and A. Y. Smirnov, *Probing large extra dimensions with neutrinos*, Nucl. Phys. B **563**, 63–81 (1999) doi:10.1016/S0550-3213(99)00574-X [arXiv:hep-ph/9904211 [hep-ph]].
[63] P. A. N. Machado, H. Nunokawa, and R. Zukanovich Funchal, *Testing for Large Extra Dimensions with Neutrino Oscillations*, Phys. Rev. D **84**, 013003 (2011) doi:10.1103/PhysRevD.84.013003 [arXiv:1101.0003 [hep-ph]].
[64] D. V. Forero, C. Giunti, C. A. Ternes, and O. Tyagi, *Large extra dimensions and neutrino experiments*, Phys. Rev. D **106**, no.3, 035027 (2022) doi:10.1103/PhysRevD.106.035027 [arXiv:2207.02790 [hep-ph]].
[65] A. Lukas, P. Ramond, A. Romanino, and G. G. Ross, *Solar neutrino oscillation from large extra dimensions*, Phys. Lett. B **495**, 136–146 (2000) doi:10.1016/S0370-2693(00)01206-5 [arXiv:hep-ph/0008049 [hep-ph]].
[66] A. Lukas, P. Ramond, A. Romanino, and G. G. Ross, *Neutrino masses and mixing in brane world theories*, JHEP **04**, 010 (2001) doi:10.1088/1126-6708/2001/04/010 [arXiv:hep-ph/0011295 [hep-ph]].
[67] M. Carena, Y. Y. Li, C. S. Machado, P. A. N. Machado, and C. E. M. Wagner, *Neutrinos in Large Extra Dimensions and Short-Baseline ν_e Appearance*, Phys. Rev. D **96** (2017) no.9, 095014 doi:10.1103/PhysRevD.96.095014 [arXiv:1708.09548 [hep-ph]].
[68] L. A. Anchordoqui, I. Antoniadis, and J. Cunat, *The dark dimension and the standard model landscape*, Phys. Rev. D, in press [arXiv:2306.16491 [hep-ph]].
[69] L. A. Anchordoqui, I. Antoniadis, K. Benakli, J. Cunat, and D. Lüst, *Searching for neutrino-modulino oscillations at the Forward Physics Facility*, [arXiv:2308.11476 [hep-ph]].
[70] K. Benakli and A. Y. Smirnov, *Neutrino-modulino mixing*, Phys. Rev. Lett. **79**, 4314–4317 (1997) doi:10.1103/PhysRevLett.79.4314 [arXiv:hep-ph/9703465 [hep-ph]].
[71] L. A. Anchordoqui, A. Ariga, T. Ariga, W. Bai, K. Balazs, B. Batell, J. Boyd, J. Bramante, M. Campanelli, and A. Carmona, et al. *The Forward Physics Facility: Sites, experiments, and physics potential*," Phys. Rept. **968**, 1–50 (2022) doi:10.1016/j.physrep.2022.04.004 [arXiv:2109.10905 [hep-ph]].
[72] J. L. Feng, F. Kling, M. H. Reno, J. Rojo, D. Soldin, L. A. Anchordoqui, J. Boyd, A. Ismail, L. Harland-Lang, and K. J. Kelly, et al. *The Forward Physics Facility at the High-Luminosity LHC*, J. Phys. G **50**, no.3, 030501 (2023) doi:10.1088/1361-6471/ac865e [arXiv:2203.05090 [hep-ex]].
[73] H. Ooguri and C. Vafa, *Non-supersymmetric AdS and the Swampland*, Adv. Theor. Math. Phys. **21** (2017), 1787–1801 doi:10.4310/ATMP.2017.v21.n7.a8 [arXiv:1610.01533 [hep-th]].
[74] E. R. Harrison, *Fluctuations at the threshold of classical cosmology*, Phys. Rev. D **1**, 2726–2730 (1970) doi:10.1103/PhysRevD.1.2726
[75] Y. B. Zeldovich, *A Hypothesis, unifying the structure and the entropy of the universe*, Mon. Not. Roy. Astron. Soc. **160**, 1P–3P (1972) doi:10.1093/mnras/160.1.1P
[76] N. Aghanim et al. [Planck], *Planck 2018 results VI: Cosmological parameters*, Astron. Astrophys. **641**, A6 (2020) [erratum: Astron. Astrophys. **652**, C4 (2021)] doi:10.1051/0004-6361/201833910 [arXiv:1807.06209 [astro-ph.CO]].
[77] B. Ratra, *Restoration of spontaneously broken continuous symmetries in de Sitter space-time*, Phys. Rev. D **31** (1985), 1931–1955 doi:10.1103/PhysRevD.31.1931.
[78] I. Antoniadis, J. Cunat, and A. Guillen, *Cosmological perturbations from five-dimensional inflation*, [arXiv:2311.17680 [hep-ph]].
[79] P. A. R. Ade et al. [BICEP and Keck], *Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season*, Phys. Rev. Lett. **127** (2021) no.15, 151301 doi:10.1103/PhysRevLett.127.151301 [arXiv:2110.00483 [astro-ph.CO]].
[80] M. Tristram, A. J. Banday, K. M. Górski, R. Keskitalo, C. R. Lawrence, K. J. Andersen, R. B. Barreiro, J. Borrill, L. P. L. Colombo and H. K. Eriksen, et al. *Improved limits on the tensor-to-scalar ratio using BICEP and Planck data*, Phys. Rev. D **105** (2022) no.8, 083524 doi:10.1103/PhysRevD.105.083524 [arXiv:2112.07961 [astro-ph.CO]].
[81] N. Arkani-Hamed, L. J. Hall, D. Tucker-Smith, and N. Weiner, *Solving the hierarchy problem with exponentially large dimensions*, Phys. Rev. D **62** (2000), 105002 doi:10.1103/PhysRevD.62.105002 [arXiv:hep-ph/9912453 [hep-ph]].
[82] N. Arkani-Hamed, S. Dubovsky, A. Nicolis, and G. Villadoro, *Quantum horizons of the standard model landscape*, JHEP **06** (2007), 078 doi:10.1088/1126-6708/2007/06/078 [arXiv:hep-th/0703067 [hep-th]].
[83] E. Abdalla et al., *Cosmology intertwined: A review of the particle physics, astrophysics, and cosmology associated with the cosmological tensions and anomalies*, JHEAp **34**, 49–211 (2022) doi:10.1016/j.jheap.2022.04.002 [arXiv:2203.06142 [astro-ph.CO]].
[84] O. Akarsu, E. Di Valentino, S. Kumar, R. C. Nunes, J. A. Vazquez, and A. Yadav, Λ_s*CDM model: A promising scenario for alleviation of cosmological tensions*, [arXiv:2307.10899 [astro-ph.CO]].
[85] Ö. Akarsu, J. D. Barrow, L. A. Escamilla, and J. A. Vazquez, *Graduated dark energy: Observational hints of a spontaneous sign switch in the cosmological constant*, Phys. Rev. D **101** (2020) no.6, 063528 doi:10.1103/PhysRevD.101.063528 [arXiv:1912.08751 [astro-ph.CO]].
[86] S. A. Adil, U. Mukhopadhyay, A. A. Sen, and S. Vagnozzi, *Dark energy in light of the early JWST observations: case for a negative cosmological constant?*, JCAP **10** (2023), 072 doi:10.1088/1475-7516/2023/10/072 [arXiv:2307.12763 [astro-ph.CO]].
[87] L. A. Anchordoqui, I. Antoniadis, and D. Lust, *Anti-de Sitter $\to$ de Sitter transition driven by Casimir forces and mitigating tensions in cosmological parameters*, [arXiv:2312.12352 [hep-th]].