Peccei-Quinn Mechanism in the 3-3-1 Model

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Abstract

The axion which is very attractive to experimental searches, is a popular topic in the modern physics. This is arised from the spontaneous breaking of the the global $U(1)_{Q_A}$ symmetry that was implemented by Peccei-Quinn (PQ) to solve the Strong CP Problem. Among various versions of 3-3-1 models, there is a special version in which the PQ charge operator can be constructed. It is based on the $SU(3)_C \times SU(3)_L \times U(1)_N$(3-3-1) gauge group.

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1. INTRODUCTION

It is well-known that the PQ mechanism [1, 2] has been constructed to solve the Strong CP Problem. The axion has generated not only much interest but also some confusion. The new development in this direction which is called GKR, is redefinition in 1986 by Georgi, Kaplan and Randall [3] as follows

$$\Phi = R(x)e^{i\frac{a(x)}{f_a}x_\phi}, \tag{1}$$

where x_ϕ and f_a are the PQ charges of ϕ and axion decay constant, respectively. The worthy note here is that f_a must be very large: $f_a \geq 10^{10}$ GeV. And the factor a/f_a ensures a solution to the strong CP puzzle.

The PQ formalism is considered in many versions of the Beyond Standard Models (BSMs). Among them, the models based on the 3-3-1 gauge group [4, 5, 6, 7, 8, 9, 10, 11, 12, 13] have some intriguing properties such as:

(i) Due to the fact that one generation of quarks transforms differently from two other ones, the anomaly free condition leads to the number of generation must be the multiple of color number. On the other hand, the QCD asymptotic free requires the number of quarks must be not bigger than five. Combining these two conditions, one gets the number of quark generation is three. The same reason can explain why top quark is so heavy (175 GeV).

(ii) The PQ symmetry is automatically fulfilled in the 3-3-1 models [14].

(iii) The models provide electric charge quantization [15, 16].

In the frameworks of the 3-3-1 models, the PQ formalism has been studies more than two decades ago [17, 18, 19, 20, 21]. However, in these works, the main ingredient singlet ϕ is expanded as an ordinary complex scalar field, i.e., in the sum of CP-even and CP-odd components. This results to the mixing between axion and other CP-odd scalars. The axion is the pure imaginary part of ϕ only when $v_\phi \gg v_\chi$ with v_ϕ is the vacuum expectation value (VEV) responses to the breaking of $SU(3)_L$ down to SM subgroup. In [19], the PQ symmetry was considered in two main versions: the minimal 3-3-1 model [4, 5, 6, 7] and the version with right-handed neutrinos [8, 9, 10, 11, 12, 13].

A new development in this direction was considered five years ago in [22], where the discrete symmetry $Z_{11} \times Z_2$ is imposed, and Majorana right-handed neutrinos are introduced. To provide masses for these neutrinos, a complex scalar transforming as a $SU(3)_L$ singlet is added. Consequently, the model also contains a heavy CP-even scalar with mass in the range of 10^{10} GeV and this field might be an inflaton. However, the work in [22] still contains some flawed points in the particles' assignment of the Z_2 symmetry, the incorrect mixing mass matrix of the CP-odd scalars, and still an absence of the identification of the Standard Model-like Higgs boson. These mentioned problems have been solved in [23]. In this work, using Euler rotation method, the correct mixing matrix of CP-odd sector has been performed. For the CP-even sector, applying the Hatree-Fock approximation, the 4×4 matrix has been diagonalized. As the result, the model contains the expected inflaton with mass around 10^{11} GeV, one heavy scalar with mass at TeV scale labeled by H_χ, one scalar with mass at the electroweak (EW) scale (h_5), and of course the SM-like Higgs boson (h).

Note that in the versions mentioned above, the particle whose name was axion is indeed an axion-like particle, because it does not arise as a phase of the singlet scalar. The real axion model has been recently constructed [24]. Based on PQ charge assignment, the operator in terms of diagonal generators has been constructed as follows [24]

$$Q_A = 2T_3 - \frac{2}{\sqrt{3}}T_8 + \mathcal{X}_{PQ}. \tag{2}$$

This obtained model is really beautiful. And the aim of this paper is to present in details the formula of PQ charge operator which has been not seen before and the couplings of axion with fermions, scalar and gauge bosons.

2. THE 3-3-1 MODEL WITH INFLATION

The spectrum of the particles in the model is as below [22, 23]:

(i) Leptons are in triplets

$$\begin{gathered} f^a_L = \begin{pmatrix} \nu^a_L \\ l^a_L \\ \nu^{ca}_L \end{pmatrix} \sim (1,3,-1/3,1/3), \quad l^a_R \sim (1,1,-1,1), \\ N_{aR} \sim (1,1,0,-1) \text{ Majorana RH neutrino,} \end{gathered} \tag{3}$$

where $a = 1,2,3$ and the numbers in parentheses are the quantum numbers for $SU(3)_C$, $SU(3)_L$, $U(1)_N$, and $U(1)_{Q_A}$, respectively.

(ii) Two quark generations are in antitriplets and one is in triplet

$$\begin{aligned} Q_{\alpha L} &= \begin{pmatrix} d_{\alpha L} \\ -u_{\alpha L} \\ D_{\alpha L} \end{pmatrix} \sim (3,\bar{3},0,-1/3), \quad \alpha = 1,2, \\ d_{\alpha R}, D_{\alpha R} &\sim (3,1,-1/3,1), \\ Q_{3L} &= \begin{pmatrix} t \\ b \\ T \end{pmatrix}_L \sim (3,3,1/3,1/3), \\ t_R, T_R &\sim (3,1,2/3,-1). \end{aligned} \tag{4}$$

(iii) For the spontaneous symmetry breaking (SSB), we need three Higgs triplets and one singlet

$$\begin{aligned} \rho &= \begin{pmatrix} \rho^+_1 \\ \rho^0_2 \\ \rho^+_3 \end{pmatrix} \sim (1,3,2/3,-2/3), \\ \eta &= \begin{pmatrix} \eta^0_1 \\ \eta^-_2 \\ \eta^0_3 \end{pmatrix} \sim (1,3,-1/3,4/3), \\ \chi &= \begin{pmatrix} \chi^0_1 \\ \chi^-_2 \\ \chi^0_3 \end{pmatrix} \sim (1,3,-1/3,4/3), \quad \phi \sim (1,1,0,2). \end{aligned} \tag{5}$$

Pay attention that η and χ have the same quantum number, but have different VEV structures in their neutral components:

$$\begin{gathered} \langle\rho\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \quad \langle\eta\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \\ \langle\chi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}, \quad \langle\phi\rangle = \frac{1}{\sqrt{2}}v_\phi. \end{gathered} \tag{6}$$

Since lepton and anti-lepton lie in the same triplet, the lepton number is not conserved. The new conserved value should have the form [25]

$$L = \alpha T_3 + \beta T_8 + \mathcal{L}. \tag{7}$$

2.1. Higgs Sector

The full potential of the model under consideration has the form [22, 23, 24] as below:

$$\begin{aligned} V_{\text{tot}} = & \mu_\chi^2 \chi^\dagger \chi + \mu_\rho^2 \rho^\dagger \rho + \mu_\eta^2 \eta^\dagger \eta + \mu_\phi^2 \phi^* \phi + \lambda_1 \left(\chi^\dagger \chi\right)^2 + \lambda_2 \left(\eta^\dagger \eta\right)^2 \\ & + \lambda_3 \left(\rho^\dagger \rho\right)^2 + \lambda_4 \left(\chi^\dagger \chi\right)\left(\eta^\dagger \eta\right) + \lambda_5 \left(\chi^\dagger \chi\right)\left(\rho^\dagger \rho\right) + \lambda_6 \left(\eta^\dagger \eta\right)\left(\rho^\dagger \rho\right) \\ & + \lambda_7 \left(\chi^\dagger \eta\right)\left(\eta^\dagger \chi\right) + \lambda_8 \left(\chi^\dagger \rho\right)\left(\rho^\dagger \chi\right) + \lambda_9 \left(\eta^\dagger \rho\right)\left(\rho^\dagger \eta\right) \\ & + \lambda_{10} \left(\phi^* \phi\right)^2 + \lambda_{11} \left(\phi^* \phi\right)\left(\chi^\dagger \chi\right) + \lambda_{12} \left(\phi^* \phi\right)\left(\rho^\dagger \rho\right) \\ & + \lambda_{13} \left(\phi^* \phi\right)\left(\eta^\dagger \eta\right) + \left(\lambda_\phi \epsilon^{ijk} \eta_i \rho_j \chi_k \phi^* + \text{H.c}\right). \end{aligned} \tag{8}$$

Expansions of the scalar fields

$$\begin{aligned} \chi^T &= \left(\chi_1^0, \chi_2^-, \chi_3^0\right) \sim \left(1, 3, -\frac{1}{3}\right), \quad \eta^T = \left(\eta_1^0, \eta_2^-, \eta_3^0\right) \sim \left(1, 3, -\frac{1}{3}\right), \\ \rho^T &= \left(\rho_1^+, \rho_2^0, \rho_3^+\right) \sim \left(1, 3, \frac{2}{3}\right), \quad \phi = \frac{1}{2}\left(v_\phi + R_\phi\right) e^{i\frac{a}{f_a}} \sim (1, 1, 0), \end{aligned} \tag{9}$$

where $\tan\theta_{PQ} = \frac{I_\phi}{v_\phi + R_\phi}$: I_ϕ? There is not I_ϕ in ϕ.

Let us expand these scalar fields around their VEVs.

$$\begin{aligned} \rho_2^0 &= \frac{1}{\sqrt{2}}\left(v_\rho + R_\rho + iI_\rho\right), \quad \eta_1^0 = \frac{1}{\sqrt{2}}\left(v_\eta + R_\eta^1 + iI_\eta^1\right), \\ \chi_3^0 &= \frac{1}{\sqrt{2}}\left(v_\chi + R_\chi^3 + iI_\chi^3\right), \quad \phi = \frac{1}{2}\left(v_\phi + R_\phi\right) e^{i\frac{a}{f_a}}. \end{aligned} \tag{10}$$

Substitution of (10) into (8) leads to the following constraints at the tree level

$$\begin{aligned} \mu_\rho^2 + \lambda_3 v_\rho^2 + \frac{\lambda_5}{2} v_\chi^2 + \frac{\lambda_6}{2} v_\eta^2 + \frac{\lambda_{12}}{2} v_\phi^2 + \frac{A}{2v_\rho^2} &= 0, \\ \mu_\eta^2 + \lambda_2 v_\eta^2 + \frac{\lambda_4}{2} v_\chi^2 + \frac{\lambda_6}{2} v_\rho^2 + \frac{\lambda_{13}}{2} v_\phi^2 + \frac{A}{2v_\eta^2} &= 0, \\ \mu_\chi^2 + \lambda_1 v_\chi^2 + \frac{\lambda_4}{2} v_\eta^2 + \frac{\lambda_5}{2} v_\rho^2 + \frac{\lambda_{11}}{2} v_\phi^2 + \frac{A}{2v_\chi^2} &= 0, \\ \mu_\phi^2 + \lambda_{10} v_\phi^2 + \frac{\lambda_{11}}{2} v_\chi^2 + \frac{\lambda_{12}}{2} v_\rho^2 + \frac{\lambda_{13}}{2} v_\eta^2 + \frac{A}{2v_\phi^2} &= 0, \end{aligned} \tag{11}$$

where $A \equiv \lambda_\phi v_\phi v_\chi v_\eta v_\rho$.

The VEV v_ϕ is responsible for the PQ symmetry breaking. The VEV v_χ breaks $SU(3)_L \times U(1)_N$ to the SM group. Two others v_ρ, v_η are needed for the usual $U(1)_Q$ symmetry. Hence, it follows $v_\phi \gg v_\chi \gg v_\rho, v_\eta$. The result in [23] is summarized as follows:

(1) One heavy field with mass in the range of $\mathcal{O}(10^{11})$ GeV and associated with the singlet ϕ being identified to inflaton Φ.
(2) One SM-like Higgs boson h with mass $\sim$125 GeV.
(3) Two remain fields include one heavy with mass at TeV scale (H_χ) and another with mass at electroweak scale (h_5).

In the limit $v_\phi \gg v_\chi \gg v_\rho \gg v_\eta$, one has

$$\begin{aligned} \eta &\simeq \begin{pmatrix} \frac{1}{\sqrt{2}}\left(v_\eta + h_5 + iA_5\right) \\ H_1^- \\ G_{X^0} \end{pmatrix}, \quad \chi \simeq \begin{pmatrix} \chi_1^0 \\ G_{Y^-} \\ \frac{1}{\sqrt{2}}\left(v_\chi + H_\chi + iG_{Z'}\right) \end{pmatrix}, \\ \rho &\simeq \begin{pmatrix} G_{W^+} \\ \frac{1}{\sqrt{2}}\left(v_\rho + h + iG_Z\right) \\ H_2^+ \end{pmatrix}, \quad \phi = \frac{1}{\sqrt{2}}\left(v_\phi + \Phi\right) e^{-i\frac{a}{f_a}}. \end{aligned} \tag{12}$$

Notice:

(i) A_5 new CP-odd scalar
(ii) χ_1^0-bilepton DM [26].

2.2. PQ Charge in the 3-3-1 Model with Inflation

For chiral fermions

$$\begin{aligned} f_L \to f_L' &= e^{-i\left(\frac{c_f}{2f_a}\right)a} f_L, \quad \bar{f}_L \to \bar{f}_L' = \bar{f}_L e^{i\left(\frac{c_f}{2f_a}\right)a}, \\ f_R \to f_R' &= e^{i\left(\frac{c_f}{2f_a}\right)a} f_R, \quad \bar{f}_R \to \bar{f}_R' = \bar{f}_R e^{-i\left(\frac{c_f}{2f_a}\right)a}, \end{aligned} \tag{13}$$

where c_f is PQ charge of fermion and $f_a \sim 10^{11}$ GeV is the axion decay constant relating to the symmetry breaking scale of the $U(1)_{PQ} \sim U(1)_{Q_A}$ global group.

The PQ charges of fermions are given as follows [22]:

$$\begin{aligned} c_u &= c_T = -c_d = -c_D = c_l = -c_{lR} \\ &= -c_\nu = c_{\nu_R} = -c_N \equiv R. \end{aligned} \tag{14}$$

Here R is a non-zero integer. $|R| = 1 \Rightarrow R = 1$. Note that all charged scalars do not have PQ charge and the singlet ϕ must carry PQ charge.

2.3. Formula of PQ Charge Operator

The PQ charges given in Table allow us to write some nice formula as generalized lepton number.

Let us write PQ charge operator as a linear combination of diagonal operators as follows (for left-handed fermions sitting in non-singlets).

$$Q_A = \alpha T_3 + \beta T_8 + \delta \mathcal{X}_{PQ}. \tag{15}$$

Applying for a $SU(3)_L$ triplet Q_3, one gets

$$\alpha = +2, \quad \beta = -\frac{2}{\sqrt{3}}, \quad \delta \mathcal{X}_{PQ}\left(Q_3\right) = +\frac{1}{3}. \tag{16}$$

Assuming $\delta = 1$, one gets $\mathcal{X}_{PQ}(Q_3) = +\frac{1}{3}$. Hence

$$Q_A = 2T_3 - \frac{2}{\sqrt{3}} T_8 + \mathcal{X}_{PQ}. \tag{17}$$

For all fermion triplets, one has $\mathcal{X}_{PQ}(\mathbf{3}) = \frac{1}{3}$.

Note that the above formula is applicable for left-handed ferions, for right-handed fermions just take opposite sign.

For scalars

$$\mathcal{X}_{PQ}(\chi, \eta) = \frac{4}{3}, \quad \mathcal{X}_{PQ}(\rho) = -\frac{2}{3}. \tag{18}$$

The PQ charge of the singlet ϕ follows from Yukawa coupling and equals 2.

Let us connect PQ charge with electric one being as follows:

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + N. \tag{19}$$

From equations (17) and (19), it follows

$$Q_A = 2Q + \mathcal{X}_{PQ} - 2N. \tag{20}$$

For singlets, their values are given from Yukawa interactions as follows:

$$\mathcal{X}_{PQ}\left(N_R\right) = 1, \quad \mathcal{X}_{PQ}\left(f_R\right) = -Q_A\left(f_L\right), \quad \mathcal{X}_{PQ}(\phi) = 2. \tag{21}$$

For the minimal 3-3-1 model, PQ symmetry may be not suitable because of the Landau pole around 5 TeV.

Note that electric charges of up and down elements in the same (anti)triplet differ by one unit, while their PQ charges by two. That is why the factor 2 appears in equation (18). Compering with equation (9) in [21], one can see that our formula is much better.

If the singlet scalar has a non-zero VEV, it means that the PQ symmetry is broken, because

$$Q_A \langle \phi \rangle = \frac{2}{\sqrt{2}} v_\phi \neq 0. \tag{22}$$

The symmetry breaking of the model under consideration is in three steps by following scheme:

$$\begin{array}{c} SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_{11} \otimes Z_2 \otimes U(1)_{Q_A} \\ \downarrow v_\phi \\ SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_2 \\ \downarrow v_\chi \\ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\ \downarrow v_\eta, v_\rho \\ SU(3)_C \otimes U(1)_Q. \end{array} \tag{23}$$

3. AXION COUPLINGS

3.1. Anomalous Axion Gauge Boson Couplings

The gauge boson self-couplings in the 3-3-1 models were calculated in [27, 28]. To understand these couplings, let us denote W to be gauge boson of the group $SU(3)$. Then the *electric* and *magnetic* fields are as follows:

$$\begin{aligned} W^a_{0\nu} &\equiv E^a_\nu, \quad \nu = 1,2,3 = i;\ a = 1,2,\ldots,8, \\ B^a_k &\equiv \epsilon_{ijk} W^a_{ij}, \quad i \neq j \neq k\ \ (j,k = 1,2,3). \end{aligned} \tag{24}$$

Then the Lagrangian of gauge bosons are follows:

$$\begin{aligned} \mathcal{L}_{\text{gauge}} \supset W_{a\mu\nu} W^{a\mu\nu} &= 2\left(W_{a0\nu} W^{a0\nu} + W_{aij} W^{aij}\right) \\ &= 2\left(E_{ai} E^{ai} + B_{ak} B^{ak}\right) = 2\left(\mathbf{E}^2 + \mathbf{B}^2\right). \end{aligned} \tag{25}$$

In the 3-3-1 models, this part contains triple and quartic gauge boson couplings which have been presented in [27, 28].

Now we consider a part with the dual tensor

$$\mathcal{L}_\theta \supset \frac{1}{2}\epsilon^{\mu\nu\alpha\beta} W_{a\mu\nu} W^a_{\alpha\beta} = 4!\epsilon^{0i\alpha\beta} W_{a0i} W^a_{\alpha\beta} = 4!\epsilon^{0ijk} W_{a0i} W^a_{jk} = 4!\left(E_{ai} B^{ai}\right) = 4!\mathbf{E}\cdot\mathbf{B} \tag{26}$$

$$= 4!\epsilon^{0ijk}\left[\partial_0 W^a_i - \partial_i W^a_0 + g f_{abc} W^b_0 W^c_i\right]\left[\partial_j W^a_k - \partial_k W^a_j + g f_{ade} W^d_j W^e_k\right]. \tag{27}$$

Here

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f_{abc} W^b_\mu W^c_\nu. \tag{28}$$

Note that in equation (28), *four space-time indexes are different.* Then, $G\tilde{G}$ is violated by $x_o \to -x_0$ (T inverse) or $x_i \to -x_i$ (Parity symmetry). Since CPT invariance, one has CP violation or CT violation.

The the 3-3-1 model with three gauge subgroups, the anomalous couplings have the form [29]

$$\mathcal{L}_{ag} = c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{2f_a} G\tilde{G} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{2f_a} W\tilde{W} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{2f_a} B\tilde{B}. \tag{29}$$

Using equation (28) and replace W^a with physical fields given by the following transformations

$$\begin{aligned} W^3_\mu &= s_W A_\mu + c_W Z_\mu, \\ W^8_\mu &= -\frac{s_W}{\sqrt{3}} A_\mu + \frac{s_W}{\sqrt{3}} \tan_W Z_\mu + \tan_W s_W Z'_\mu, \\ B_\mu &= \sqrt{\frac{3-4s_W^2}{3c_W^2}} A_\mu - \sqrt{\frac{3-4s_W^2}{3c_W^2}} s_W Z_\mu + \tan_W Z'_\mu. \end{aligned} \tag{30}$$

3.2. Axion-Fermion Couplings

Let us begin with the kinetic term of fermion as follows:

$$\mathcal{L}^0_f = \frac{i}{2}\bar{f}\gamma^\mu \overleftrightarrow{\partial_\mu} f = \frac{i}{2}\left(\bar{f}\gamma^\mu\partial_\mu f - \partial_\mu\bar{f}\gamma^\mu f\right). \tag{31}$$

The axion-fermion derivative couplings have the form

$$\begin{aligned}\mathcal{L}_{(f-a)} = \left(\frac{1}{f_a}\right)\partial_\mu a \Big[&\bar{d}\mathbf{c}_d\gamma^\mu\gamma_5 d + \bar{u}\mathbf{c}_u\gamma^\mu\gamma_5 u + \bar{T}\mathbf{c}_T\gamma^\mu\gamma_5 T + \bar{D}_\alpha\mathbf{c}_{D_\alpha}\gamma^\mu\gamma_5 D_\alpha \\ &+ \bar{l}\mathbf{c}_l\gamma^\mu\gamma_5 l + I_\nu\bar{\nu}_a\mathbf{c}_\nu\gamma^\mu\gamma_5\nu_a + \frac{1}{2}\bar{N}_a\mathbf{c}_{N_a}\gamma^\mu P_R N_a\Big]. \end{aligned} \tag{32}$$

For the coefficients in equation (32) ($\mathbf{c}_f,\ f = d, u, \ldots, N_R$), one has to count the number of color, flavor indexes and PQ charge $Q_A(f)$. It is interesting to note that the quantity $\frac{1}{f_a} \propto 10^{-11}$ is the same as the value of the Yukawa coupling responsible for proton instability arising in the supersymmetric 3-3-1 model [30].

Our result coincides with that in [31].

3.3. *Axion-Scalar Couplings*

We continue to the scalar sector. For a complex scalar φ, its Lagrangian is

$$\begin{aligned}\mathcal{L}_\varphi &= (D^\mu\varphi)^\dagger D_\mu\varphi = \left[\left(\partial_\mu - iP^\varphi_\mu\right)\varphi\right]^\dagger (\partial^\mu - iP^{\varphi\mu})\,\varphi \\ &= \partial^\mu\varphi^\dagger\partial_\mu\varphi - i\partial_\mu\varphi^\dagger P^{\varphi\mu}\varphi + i\varphi^\dagger P^{\varphi\mu}\partial_\mu\varphi + \varphi^\dagger P^\varphi_\mu P^{\varphi\mu}\varphi \\ &\equiv \mathbf{A1} + \mathbf{A2} + \mathbf{A3},\end{aligned} \tag{33}$$

where the model under consideration predicts the quartic couplings of two axions with two *neutral* scalars and triple couplings of one axion to two neutral scalars, namely

$$\begin{aligned}\mathbf{A1} = &\text{ kinetic term of } H + \left(\frac{1}{f_a}\right)^2\partial_\mu a\partial^\mu a\left(\sum_{H=\eta_1^0,\eta_3^0,\rho_2^0}^{\chi_1^0,\chi_3^0,\phi} H^* H\right) \\ &- i\left(\frac{x_\varphi}{2f_a}\right)\partial^\mu a\sum_{D=\eta,\chi}^{K=\rho^0\phi^0}\left[D^*\overset{\leftrightarrow}{\partial_\mu} D - K^*\overset{\leftrightarrow}{\partial_\mu} K\right].\end{aligned} \tag{34}$$

Next, the second term in equation (33) is

$$\mathbf{A2} = -i\left(\partial^\mu\varphi^\dagger P^\varphi_\mu\varphi - \varphi^\dagger P^\varphi_\mu\partial^\mu\varphi\right) + 2\left(\frac{x_\varphi}{2f_a}\right)(\partial^\mu a)\times\left[\varphi^\dagger P^\varphi_\mu\varphi\right]. \tag{35}$$

The first term in equation (35) is ordinary one in the 3-3-1 model without axion's participation. The second term is quartic couplings of axion, gauge and two scalar bosons.

There are no new coupling in **A3**, or other word speaking, **A3** contains ordinary couplings of scalars to gauge bosons.

In summary, new couplings of axion with neutral scalars are (including operators of up to dimension 5)

$$\begin{aligned}\mathcal{L}_{(aS)} = &\left(\frac{1}{f_a}\right)^2\partial_\mu a\partial^\mu a\left(\sum_{H=\eta_1^0,\eta_3^0,\rho_2^0}^{\chi_1^0,\chi_3^0,\phi} H^* H\right) \\ &- i\left(\frac{x_\varphi}{2f_a}\right)(\partial^\mu a)\sum_{D=\eta,\chi}^{K=\rho^0\phi^0}\left[D^*\overset{\leftrightarrow}{\partial_\mu} D - K^*\overset{\leftrightarrow}{\partial_\mu} K\right] \\ &+ 2\left(\frac{x_\varphi}{2f_a}\right)(\partial^\mu a)\,H^\dagger P^H_\mu H \\ \equiv\ & L(aaHH) + L(aHH) + L(aGHH),\end{aligned} \tag{36}$$

where G is labeled for gauge bosons. For more detail of axion couplings, the reader is referred to [29, 32, 33, 34].

3.4. *Interactions of Axion with Scalar and Gauge Bosons*

We proceed now explicit form of the above terms. Note that in equation (36), there are not only couplings of axion to scalar bosons, but also to gauge bosons. Let us consider quartic couplings

$$\begin{aligned}\mathcal{L}(aaHH) = \frac{1}{2}\left(\frac{1}{f_a}\right)^2\partial_\mu a\partial^\mu a\Big[&\left(v_\eta^2 + v_\rho^2 + v_\chi^2 + v_\phi^2\right) + 2\left(v_\eta R^1_\eta + v_\rho R_\rho + v_\chi R^3_\chi + v_\phi R_\phi\right) \\ &+ \left(R^1_\eta\right)^2 + \left(I^1_\eta\right)^2 + (R_\rho)^2 + (I_\rho)^2 + \left(R^3_\chi\right)^2 + \left(I^3_\chi\right)^2 + (R_\phi)^2 + 2\left(\eta_3^{0*}\eta_3^0 + \chi_1^{0*}\chi_1^0\right)\Big].\end{aligned} \tag{37}$$

It follows that to get a right form of axion kinetic term, the following condition is required

$$f_{PQ}^2 = v_\eta^2 + v_\rho^2 + v_\chi^2 + v_\phi^2. \tag{38}$$

Thus, one gets finally

$$\begin{aligned}\mathcal{L}(aaHH) = \frac{1}{2}\partial_\mu a \partial^\mu a + \left(\frac{1}{f_a}\right)^2 \partial_\mu a \partial^\mu a \Big\{ & v_\eta R_\eta^1 + v_\rho R_\rho + v_\chi R_\chi^3 + v_\phi R_\phi + \eta_3^{0*}\eta_3^0 + \chi_1^{0*}\chi_1^0 \\ & + \frac{1}{2}\left[\left(R_\eta^1\right)^2 + \left(I_\eta^1\right)^2 + (R_\rho)^2 + (I_\rho)^2 + \left(R_\chi^3\right)^2 + \left(I_\chi^3\right)^2 + (R_\phi)^2\right]\Big\}.\end{aligned} \tag{39}$$

In equation (39), there exist triple couplings of two axions to one CP-even scalar. One of them with the coupling strength is $\partial_\mu a \partial^\mu a R_\phi = v_\phi (f_a)^{-2} \sim (f_a)^{-1}$—the coupling of two axions to inflaton. We should add this interaction for the future study, since $\propto (f_a)^{-1}$, while terms in the second line can be neglected.

We next consider triple couplings of an axion to two scalars:

$$\begin{aligned}\mathcal{L}(aHH) = & -\left(\frac{1}{f_a}\right)\partial^\mu a \left(v_\eta \partial_\mu I_\eta^1 + v_\chi \partial_\mu I_\chi^3 + v_\rho \partial_\mu I_\rho\right) \\ & - i\left(\frac{1}{f_a}\right)\partial^\mu a \left(iI_\eta^1 \partial_\mu R_\eta^1 + iI_\rho \partial_\mu R_\rho + iI_\chi^3 \partial_\mu R_\chi^3 - iR_\eta^1 \partial_\mu I_\eta^1 - iR_\chi^3 \partial_\mu I_\chi^3 - iR_\rho \partial_\mu I_\rho + \eta_3^{0*} \overleftrightarrow{\partial_\mu} \eta_3^0 + \chi_1^{0*} \overleftrightarrow{\partial_\mu} \chi_1^0\right) \\ & = \mathcal{L}_{(aA)} + \mathcal{L}_{(aSS)},\end{aligned} \tag{40}$$

where the mixing of axion with CP-odd scalars is determined as

$$\mathcal{L}_{(aA)} = -\left(\frac{1}{f_a}\right)\partial^\mu a \left(v_\eta \partial_\mu I_\eta^1 + v_\chi \partial_\mu I_\chi^3 + v_\rho \partial_\mu I_\rho\right). \tag{41}$$

The terms in equation (41) are some kind of mixing between axion and Goldstone bosons eaten by neutral gauge bosons. This unwanted mixing could be taken away by making $U(1)$ rotation [3]. Hence, one has coupling of axion to two scalar fields given below

$$\mathcal{L}_{(aSS)} = -i\left(\frac{1}{f_a}\right)\partial^\mu a \left(iI_\eta^1 \partial_\mu R_\eta^1 + iI_\rho \partial_\mu R_\rho + iI_\chi^3 \partial_\mu R_\chi^3 - iR_\eta^1 \partial_\mu I_\eta^1 - iR_\chi^3 \partial_\mu I_\chi^3 - iR_\rho \partial_\mu I_\rho + \eta_3^{0*} \overleftrightarrow{\partial_\mu} \eta_3^0 + \chi_1^{0*} \overleftrightarrow{\partial_\mu} \chi_1^0\right). \tag{42}$$

We close this part by considering

$$\begin{aligned}\mathcal{L}(aGHH) = & 2\left(\frac{g}{f_a}\right)\partial^\mu a \left\{\left(W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)v_\eta^2 - \left(W_{3\mu} - \frac{1}{\sqrt{3}}W_{8\mu} - \frac{2}{3}t\sqrt{\frac{2}{3}}B_\mu\right)v_\rho^2 - \left(\frac{2}{\sqrt{3}}W_{8\mu} + \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)v_\chi^2\right\} \\ & + 2\left(\frac{g}{f_a}\right)\partial^\mu a \left\{\left(W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right) \times \left[2v_\eta R_\eta^1 + \left(R_\eta^1\right)^2 + \left(I_\eta^1\right)^2\right]\right. \\ & + \left(-W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} + \frac{2}{3}t\sqrt{\frac{2}{3}}B_\mu\right) \times \left[2v_\rho R_\rho^2 + (R_\rho)^2 + (I_\rho)^2\right] \\ & + \left(-\frac{2}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)\left[2v_\chi R_\chi^3 + \left(R_\chi^3\right)^2 + \left(I_\chi^3\right)^2\right] \\ & \left. + \left(-\frac{2}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)\eta_3^{0*}\eta_3^0 + \left(W_{3\mu} + \frac{1}{\sqrt{3}}W_{8\mu} - \frac{1}{3}t\sqrt{\frac{2}{3}}B_\mu\right)\chi_1^{0*}\chi_1^0\right\} \\ & = \mathcal{L}_{(aW)} + \mathcal{L}_{aGHH},\end{aligned} \tag{43}$$

where the first term shows the mixing terms of axion with weak gauge bosons

$$\mathcal{L}_{(aW)} = 2\left(\frac{g}{f_a}\right)\partial^\mu a\left[\left(\frac{Z_\mu}{c_W} + Z'_\mu\frac{1-t_W^2}{\sqrt{3-t_W^2}}\right)v_\eta^2 + \left(\frac{Z'_{2\mu}}{c_W^2\sqrt{3-t_W^2}} - \frac{Z_\mu}{c_W}\right)v_\rho^2 + \left(-\frac{2Z'_\mu}{\sqrt{3-t_W^2}}\right)v_\chi^2\right] = vev^2\cdot\partial_\mu a Z^\mu, \tag{44}$$

$$\begin{aligned}\mathcal{L}_{aGHH} = 2\left(\frac{g}{f_a}\right)\partial^\mu a &\left[\left(\frac{Z_\mu}{c_W} + Z'_\mu\frac{1-t_W^2}{\sqrt{3-t_W^2}}\right)\times\left[2v_\eta R_\eta^1 + \left(R_\eta^1\right)^2 + \left(I_\eta^1\right)^2\right]\right.\\
&+\left(\frac{Z'_{2\mu}}{c_W^2\sqrt{3-t_W^2}} - \frac{Z_\mu}{c_W}\right)\left[2v_\rho R_\rho^2 + \left(R_\rho\right)^2 + \left(I_\rho\right)^2\right]\\
&+\left(-\frac{2Z'_\mu}{\sqrt{3-t_W^2}}\right)\left[2v_\chi R_\chi^3 + \left(R_\chi^3\right)^2 + \left(I_\chi^3\right)^2\right]\\
&\left.+\left(-\frac{2Z'_\mu}{\sqrt{3-t_W^2}}\right)\eta_3^{0*}\eta_3^0 + \left(\frac{Z_\mu}{c_W} + Z'_\mu\frac{1-t_W^2}{\sqrt{3-t_W^2}}\right)\chi_1^{0*}\chi_1^0\right]\\
= \text{Couplings}(aGHH),&\end{aligned} \tag{45}$$

where $t = \frac{3\sqrt{2}t_W}{\sqrt{3-t_W^2}}$. We also use the limit $v_\chi^2 \gg v^2$, which results in the following relations between physical basis (A_μ, Z_μ, Z'_μ) and the original gauge boson states [11]: equation (45), there are couplings, up to dimension 5, between axion, gauge boson with one scalar or two scalars.

3.5. *Total Axion Lagrangian*

The total part concerned to axion is given below

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu a\partial^\mu a - \frac{1}{2}m_{ao}^2 a^2 + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{2f_a}G\tilde{G} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{2f_a}W\tilde{W} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{2f_a}B\tilde{B} \tag{46}$$

$$+\frac{\partial^\mu a}{2f_a}\left(\sum_{f=u,d,T,D}^{l,\nu}\bar{\psi}_f c_f\gamma_\mu\gamma_5\psi + \frac{1}{2}\bar{N}_a\mathbf{c}_{N_a}\gamma^\mu P_R N_a\right) - (\bar{q}_L M_a q_R + \text{H.c.}) \tag{47}$$

$$+\left(\frac{1}{f_a}\right)^2\partial_\mu a\partial^\mu a\left(\sum_{H=\eta_1^0,\eta_3^0,\rho_2^0}^{\chi_1^0,\chi_3^0,\phi} H^*H\right) \tag{48}$$

$$-i\left(\frac{x_\varphi}{2f_a}\right)\partial^\mu a\sum_{D=\eta,\chi}^{K=\rho^0\phi^0}\left[D^*\overset{\leftrightarrow}{\partial_\mu}D - K^*\overset{\leftrightarrow}{\partial_\mu}K\right] \tag{49}$$

$$+2\left(\frac{x_\varphi}{2f_a}\right)\partial^\mu a\sum_{H=\eta_1^0,\eta_3^0,\rho_2^0}^{\chi_1^0,\chi_3^0,\phi} H^\dagger P_\mu^H H, \tag{50}$$

where $H^*H = \frac{1}{2}[(v_H + R_H)^2 + I_H^2]$.

The axion mass m_{ao} in the first line is acquired from mixing of the particle with π^0 and η pseudoscalars. The term in equation (48) provides kinematic term of axion with f_a^2 is sum over all VEVs of scalars. Beside ordinary dimension 6 terms, there are dimension 5 terms associated with VEVs in the form $(\frac{1}{f_a})^2 v_H\partial_\mu a\partial^\mu aH$. When $H = \phi$, then we have a key term proportional to $1/f_a$ only: $(\frac{1}{f_a})^2 v_\phi\partial_\mu a\partial^\mu a\Phi \sim (\frac{1}{f_a})\Phi\partial_\mu a\partial^\mu a$. The last two terms in the above expression give couplings of axion with scalar and gauge fields. The last term in equation (50) contains unwanted mixing of axion with Goldstone bosons eaten by massive Z and Z' bosons. Fortunately, this mixing can be rotated away by the $U(1)$ rotation. It is emphasized that the last term in the second line, the right handed Majorana neutrinos are very heavy with mass around 10^7 GeV.

3.6. *Elimination of Mixing between Axion and Goldstone Bosons G_Z and $G_{Z'}$*

As seen in Section 3.4, there are terms of mixing axion with Goldstone bosons G_Z and $G_{Z'}$. To avoid this trouble [3], we rotate scalar triplets by opposite $U(1)_{PQ}$ as follows:

$$\begin{aligned}
\chi &\to \chi' = e^{-i\left(\frac{a}{2f_a}\right)}\chi, \\
\eta &\to \eta' = e^{-i\left(\frac{a}{2f_a}\right)}\eta, \\
\rho &\to \rho' = e^{i\left(\frac{a}{2f_a}\right)}\rho.
\end{aligned} \tag{51}$$

Of course, the quarks have to be changed to keep Yukawa interactions invariant. Note that the singlet ϕ in unchanged. Then, equation (50) becomes *final* Lagrangian as the following

$$\begin{aligned}
\mathcal{L}_a &= \frac{1}{2}\partial_\mu a \partial^\mu a + c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{2f_a}G\tilde{G} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{2f_a}W\tilde{W} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{2f_a}B\tilde{B} \\
&+ \frac{\partial^\mu a}{2f_a}\left(\sum_{f=u,d,T,D}^{l,\nu}\bar{\psi}_f c_f \gamma_\mu\gamma_5\psi + \frac{1}{2}\bar{N}_a \mathbf{c}_{N_a}\gamma^\mu P_R N_a\right) - (\bar{q}_L M_a q_R + \text{H.c.}) \\
&+ \left(\frac{1}{2f_a}\right)^2 \partial_\mu a \partial^\mu a \left(v_\phi^2 + 2v_\phi R_\phi + R_\phi^2\right).
\end{aligned} \tag{52}$$

To have correct axion's kinetic term, equation (52) leads to condition

$$f_a = v_\phi. \tag{53}$$

The point is worth noting that the terms in the last line (doubly ($\partial_\mu a \partial^\mu a$) derivative coupling of axion to inflaton) are characteristic for the model under consideration. This coupling is dimension 5, too. As mentioned in [3], the axion has only two kinds of couplings: derivative couplings to fermions and anomalous couplings to gauge bosons $aG\tilde{G}$. It is worth noting that the new effects mainly happen in the energy region from 10^7 GeV to 10^{11} GeV, namely in the region from mass of Majorana right-handed neutrino (N_R) to mass of inflaton Φ.

4. AXION MASS IN THREE QUARK FLAVORS

For three flavors: 9 matrices of $U(3)$ group: 8 Gell-mann and one

$$\lambda_0 = \sqrt{\frac{2}{3}}\text{diag}(1,1,1), \tag{54}$$

$$K = \sum_{a=1}^{8}\lambda_a K_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \tag{55}$$

Then,

$$V = e^{i\sum_{a=1}^{8}\lambda_a\frac{K_a}{f_\pi}} = \mathbf{1}\cos\left(\frac{\mathcal{K}}{f_\pi}\right) + i\frac{\lambda_a K_a}{\mathcal{K}}\sin\left(\frac{\mathcal{K}}{f_\pi}\right), \tag{56}$$

where

$$\mathcal{K} = \sqrt{(\pi^o)^2 + 2\pi^+\pi^- + 3\eta^2 + 2K^+K^- + 2K^0\bar{K}^0}, \tag{57}$$

$$\begin{aligned}
\det K &= \left(\pi^0 + \frac{1}{\sqrt{3}}\eta\right)\left(-\pi^0 + \frac{1}{\sqrt{3}}\eta\right)\left(-\frac{2}{\sqrt{3}}\eta\right) \\
&+ \left(\sqrt{2}\pi^+\right)\left(\sqrt{2}K^0\right)\left(\sqrt{2}K^-\right) + \left(\sqrt{2}\pi^-\right)\left(\sqrt{2}\bar{K}^0\right)\left(\sqrt{2}K^+\right) \\
&- \left(2\pi^+\pi^-\right)\left(-\frac{2}{\sqrt{3}}\eta\right) - 2K^0\bar{K}^0\left(\pi^0 + \frac{1}{\sqrt{3}}\eta\right).
\end{aligned} \tag{58}$$

According to [35],

$$\begin{aligned}
M_{3q} = \text{diag}\left(m_u, m_d, m_s\right) &= \frac{m_u + m_d + m_s}{\sqrt{6}}\lambda_0 \\
&+ \frac{m_u + m_d - 2m_s}{2\sqrt{3}}\lambda_8 + \frac{m_u - m_d}{2}\lambda_3.
\end{aligned} \tag{59}$$

Hence,

$$\Rightarrow M_{3q}^{-1} = \frac{1}{m_u m_d ms} \begin{pmatrix} m_d m_s & 0 & 0 \\ 0 & m_u m_s & 0 \\ 0 & 0 & m_u m_d \end{pmatrix} = \text{diag}\left(1/m_u, 1/m_d, 1/m_s\right), \tag{60}$$

$$\text{Tr}\ \left(M_{3q}\right)^{-1} = \frac{m_u m_d + m_u m_s + m_d m_s}{m_u m_d m_s}, \tag{61}$$

also

$$\frac{M_{3q}^{-1}}{3}\text{Tr}\ \left(M_{3q}\right)^{-1} = \text{diag}\left(\frac{1}{m_u}, \frac{1}{m_d}, \frac{1}{m_s}\right) \frac{m_u m_d m_s}{(m_u m_d + m_u m_s + m_d m_s)}. \tag{62}$$

4.1. $SU(3)$ Group

For three flavors: 9 matrices of $U(3)$ group: 8 Gell-mann and one

$$\lambda_0 = \sqrt{\frac{2}{3}}\text{diag}(1,1,1), \tag{63}$$

$$K = \sum_{a=1}^{8} \lambda_a K_a = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\tilde{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}, \tag{64}$$

$$K = \sum_{a=1}^{8} \lambda_a K_a, \quad \Rightarrow K_a = \frac{1}{2}\text{Tr}\left(\lambda_a K\right), \tag{65}$$

$$\begin{aligned} K_3 &= \frac{1}{2}\text{Tr}\left(\lambda_3 K\right) = \frac{1}{2}\left[\left(\pi^0 + \frac{1}{\sqrt{3}}\eta\right) - \left(-\pi^0 + \frac{1}{\sqrt{3}}\eta\right)\right] = \pi^0, \\ K_8 &= \frac{1}{2}\text{Tr}\left(\lambda_8 K\right) = \frac{1}{2\sqrt{3}}\left[\left(\pi^0 + \frac{1}{\sqrt{3}}\eta\right) + \left(-\pi^0 + \frac{1}{\sqrt{3}}\eta\right) + \frac{4}{\sqrt{3}}\eta\right] = \sqrt{3}\eta, \\ K_1 &= \frac{1}{2}\text{Tr}\left(\lambda_1 K\right) = \frac{\pi^+ + \pi^-}{\sqrt{2}}, \\ K_2 &= \frac{1}{2}\text{Tr}\left(\lambda_2 K\right) = i\frac{-\pi^+ + \pi^-}{\sqrt{2}}, \end{aligned} \tag{66}$$

$$\begin{aligned} K_4 &= \frac{1}{2}\text{Tr}\left(\lambda_4 K\right) = \frac{K^+ + K^-}{\sqrt{2}}, \\ K_5 &= \frac{1}{2}\text{Tr}\left(\lambda_5 K\right) = i\frac{-K^+ + K^-}{\sqrt{2}}, \\ K_6 &= \frac{1}{2}\text{Tr}\left(\lambda_6 K\right) = \frac{K^0 + \tilde{K}^0}{\sqrt{2}}, \\ K_7 &= \frac{1}{2}\text{Tr}\left(\lambda_7 K\right) = i\frac{-\tilde{K}^0 + +K^0}{\sqrt{2}}. \end{aligned} \tag{67}$$

Then,

$$\mathcal{K}^2 = \sum_{i=1}^{3} K_i^2 + \sum_{i=4}^{8} K_i^2 = \pi^2 + K_{48}^2, \tag{68}$$

where

$$K_{48}^2 = \sum_{i=4}^{8} K_i^2 = 3\eta^2 + 2K^+K^- + 2K^0\tilde{K}^0. \tag{69}$$

Then,

$$V = e^{i\sum_{a=1}^{8}\lambda_a \frac{K_a}{f_\pi}} = \mathbf{1}\cos\left(\frac{\mathcal{K}}{f_\pi}\right) + i\frac{\lambda_a K_a}{\mathcal{K}}\sin\left(\frac{\mathcal{K}}{f_\pi}\right), \tag{70}$$

where

$$\mathcal{K} = \sqrt{(\pi^o)^2 + 2\pi^+\pi^- + 3\eta^2 + 2K^+K^- + 2K^0\tilde{K}^0}. \tag{71}$$

Combination of (59) and (70) yields

$$VM_{3q}^{\dagger} + M_{3q}V^{\dagger} = VM_{3q} + M_{3q}V^{\dagger} + i\frac{a}{2f_a}\left\{Q_A, M_{3q}\right\}V^{\dagger} - i\frac{a}{2f_a}V\left\{Q_A, M_{3q}\right\} + \cdots \tag{72}$$

with

$$\begin{aligned} Q_A &= \frac{M_{3q}^{-1}}{3}\mathrm{Tr}\,\left(M_{3q}\right)^{-1} \\ &= \mathrm{diag}\left(1/m_u, 1/m_d, 1/m_s\right)\frac{m_u m_d m_s}{(m_u m_d + m_u m_s + m_d m_s)}, \end{aligned} \tag{73}$$

we have

$$\begin{aligned} \mathcal{L}_{\mathrm{amass}} \supset \mathrm{Tr}\left[VM_{3q} + M_{3q}V^{\dagger}\right] &= \left[\mathbf{1}\cos\frac{\mathcal{K}}{f_\pi} + i\sum_{a=1}^{8}\lambda_a\frac{\phi_a}{\mathcal{K}}\sin\frac{\mathcal{K}}{f_\pi}\right] \\ &\times\left[\frac{m_u+m_d+m_s}{\sqrt{6}}\lambda_0 + \frac{m_u+m_d-2m_s}{2\sqrt{3}}\lambda_8 + \frac{m_u-m_d}{2}\lambda_3\right] \\ &= 2\frac{1}{3}\left(m_u+m_d+m_s\right)\cos\frac{\mathcal{K}}{f_\pi}. \end{aligned} \tag{74}$$

Using $m_\pi^2 = B_0(m_u+m_d)$, $m_{K^+}^2 = B_0(m_u+m_s)$, $m_{K^0}^2 = B_0(m_d+m_s)$, one gets

$$\begin{aligned} \mathcal{L}_{\mathrm{amass}} &\supset \frac{f_\pi^2}{4}2B_0 2\frac{1}{3}\left(m_u+m_d+m_s\right)\cos\frac{\mathcal{K}}{f_\pi} \\ &= \frac{1}{6}f_\pi^2\left(m_\pi^2 + m_{K^0}^2 + m_{K^+}^2\right)\cos\frac{\mathcal{K}}{f_\pi}, \end{aligned} \tag{75}$$

where [35]

$$m_{K^0}^2 = B_0\left(m_u+m_s\right), \quad m_{K^+}^2 = B_0\left(m_d+m_s\right). \tag{76}$$

As before

$$\mathcal{L}_{\mathrm{amass}} \supset i\frac{a}{2f_a}\left\{Q_A, M_{3q}\right\}V^{\dagger} - i\frac{a}{2f_a}V\left\{Q_A, M_{3q}\right\} = 0. \tag{77}$$

Third term

$$\begin{aligned} \mathcal{L}_{\mathrm{amass}} &\supset -2\frac{f_\pi^2}{2}B_0\left(\frac{a}{2f_a}\right)^2 \times \mathrm{Tr}\left[V\left(Q_A^2 M_{3q} + 2Q_A M_{3q} Q_A + M_{3q}Q_A^2\right)\right] \\ &= -\frac{f_\pi^2}{2}\left(\frac{a}{2f_a}\right)^2 B_0\frac{1}{\left(\mathrm{Tr}\,M_{3q}^{-1}\right)^2}\mathrm{Tr}\,\left(VM_Q^{-1}\right) \\ &= -\frac{f_\pi^2}{4}B_0\left(\frac{a}{f_a}\right)^2\left(\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right)\cos\frac{\mathcal{K}}{f_\pi}. \end{aligned} \tag{78}$$

Hence,

$$\begin{aligned} m_a^2 &= \frac{f_\pi^2}{2}B_0\left(\frac{1}{f_a}\right)^2\left(\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right)\cos\frac{\mathcal{K}}{f_\pi} \\ &\simeq \frac{1}{2}B_0\left(\frac{f_\pi}{f_a}\right)^2\left(\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right). \end{aligned} \tag{79}$$

Summary

$$\begin{aligned} \mathcal{L}_{\mathrm{amass}} &= \frac{f_\pi^2}{4}2B_0 2\frac{1}{3}\left(m_u+m_d+m_s\right)\cos\frac{\mathcal{K}}{f_\pi} \\ &= \frac{1}{6}f_\pi^2\left(m_\pi^2 + m_{K^0}^2 + m_{K^+}^2\right)\cos\frac{\mathcal{K}}{f_\pi} - \frac{m_a^2}{2}, \end{aligned} \tag{80}$$

where

$$\begin{aligned} m_a^2 &= \frac{1}{2} B_0 \left(\frac{f_\pi}{f_a}\right)^2 \left(\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right) \\ &= \frac{1}{2} m_\pi^2 \left(\frac{f_\pi}{f_a}\right)^2 \frac{1}{(m_u + m_d)} \left(\frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s}\right). \end{aligned} \tag{81}$$

This result is similar to that in [36]

$$\begin{aligned} m_a &= 4 \frac{f_\pi m_\pi}{f_a / N} \left[\frac{m_u m_d m_s}{(m_u m_d + m_u m_s + m_d m_s)(m_u + m_d)}\right]^{\frac{1}{2}} \\ &\simeq \left(1.2 \times 10^{-5} \mathrm{eV}\right) \left(\frac{10^{12}\, \mathrm{GeV}}{f_a / N}\right). \end{aligned} \tag{82}$$

5. CONCLUSIONS

In this paper, we have presented PQ formalism of the 3-3-1 model with inflation. The singlet field ϕ takes complex value everywhere, and axion is the complex phase in polar coordinates. Then the axion also appears as a phase of PQ transformations. Using the GKS formation, we have constructed PQ charge operator Q_A in terms of diagonal generators of the $SU(3)_L$ subgroup. Compering with similar result in [21], one can see that our formula is much better. In the model under consideration, in contrast to photons coupling to charged particles only, the axion does not couple to charged scalar/gauge bosons. In other words, axion is an electric chargephobic for bosons, implying that the effective couplings of axion relating to photon such as $a\gamma\gamma$, and $Za\gamma$ have one-loop contributions from only fermions. To have correct kinetic term for the axion, the PQ scale f_a is to equal VEV of the singlet scalar boson, namely $f_a = v_\phi$. The derivative couplings of axion to fermions were presented. The point is worth emphasizing that the axion has doubly derivative coupling with scalar playing the role of inflaton. This coupling increases magnitude of coupling to $1/f_a$. Also, the new effects mainly happen in the energy region from 10^7 GeV to 10^{11} GeV, namely in the region from mass of Majorana right-handed neutrino (N_R) to mass of inflaton Φ [3]. The chiral effective Lagrangian as usually provides axion mass consistent with model-independent prediction.

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