

# New Insights on Lepton Number and Dark Matter

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## Abstract

Dark matter (DM) is usually assumed to be stabilized by a symmetry, which is mostly considered to be  $Z_2$ . For example, in supersymmetry it is  $R$  parity, i.e.  $(-1)^{3B+L+2j}$ . However, it may be  $Z_n$  or  $U(1)_D$ , and derivable from generalized lepton number. In this context, neutrinos may be Majorana or Dirac, and owe their existence to dark matter, i.e. they are scotogenic.

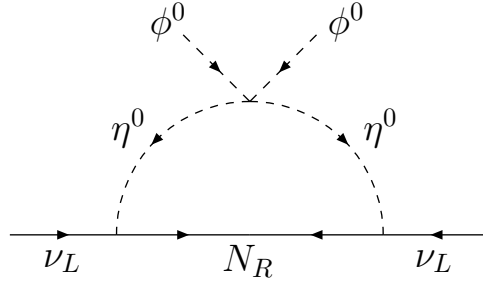
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## 1. DARK MATTER PROTOTYPES

The simplest DM model [1] is to add a real neutral singlet scalar  $S$  to the Standard Model (SM) with a new  $Z_2$  symmetry under which  $S$  is odd and all other fields are even. This symmetry is necessary because the would-be allowed term  $S\Phi^\dagger\Phi$  in the Lagrangian must be forbidden,  $\Phi$  being the SM Higgs doublet. It also forbids the possible  $S\nu_R\nu_R$  term if  $\nu_R$  is added for  $\nu_L$  to acquire a small seesaw Majorana mass. The next simplest model [2] is to add a singlet Majorana fermion  $\chi_L$  so that the term  $S\chi_L\nu_R$  is allowed.

Another DM prototype is for it to generate a radiative Majorana neutrino mass, i.e. the scotogenic mechanism. The simplest one-loop example [3] adds three Majorana neutral singlet fermions  $N_R$  and one scalar doublet  $\eta = (\eta^+, \eta^0)$  to the SM. A new  $Z_2$  symmetry is again assumed under which they are odd and all other fields are even. Hence the tree-level terms  $\bar{\nu}_L N_R \phi^0$  are forbidden, but  $\bar{\nu}_L N_R \eta^0$  are allowed. The same idea also works in some well-known three-loop models [4, 5, 6] of neutrino mass.



**FIGURE 1:** Radiative seesaw neutrino mass: the scotogenic mechanism.

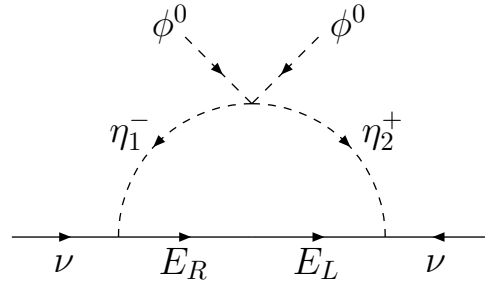
## 2. DARK PARITY FROM LEPTON PARITY

Even without supersymmetry, the factor  $(-1)^{2j}$  may be used to obtain dark parity  $\pi_D$  from lepton parity  $\pi_L = (-1)^L$ . This simple observation [7] shows that the assignment of lepton parity to new particles added to the SM would also determine the dark sector, i.e. no new  $Z_2$  symmetry is required to obtain exactly the same Lagrangian.

In the SM, under  $\pi_L$ , leptons (which are all fermions) are odd and other fields are even. In the DM prototypes,  $S$  should be assigned odd and  $\chi_L$  even, so that  $S\Phi^\dagger\Phi$  and  $S\nu_R\nu_R$  are forbidden, whereas  $S\chi_L\nu_R$  is allowed. It is clear that the previously imposed  $Z_2$  dark parity  $\pi_D$  is just  $(-1)^{2j}\pi_L$ . In the scotogenic model,  $N$  and  $\eta$  should be assigned even and odd respectively. Similar assignments are applicable as well in the KNT [4], AKS [5], and GNR [6] models.

## 3. LEPTON PARITY WITH DARK $U(1)_D$

Instead of assuming lepton parity to begin with, a more general approach is to use global  $U(1)_L$  and break it softly by two units, but with a particle content such that a dark  $U(1)_D$  symmetry remains. Add to the SM three pairs of charged fermions  $E_L \sim 0, E_R \sim 2$ , and two scalar doublets  $(\eta_1^0, \eta_1^-) \sim 1, (\eta_2^{++}, \eta_2^+) \sim -1$ , plus one scalar singlet  $\chi^0 \sim -1$ , then use the soft term  $\bar{E}_L E_R$  to break  $U(1)_L$  by two units. A scotogenic Majorana neutrino mass is obtained, but  $U(1)_D$  remains. Here  $\chi^0$  (mixing slightly with  $\tilde{\eta}^0$ ) is DM.



**FIGURE 2:** Scotogenic Majorana neutrino mass with  $U(1)_D$ .

#### 4. LEPTON NUMBER VARIANTS

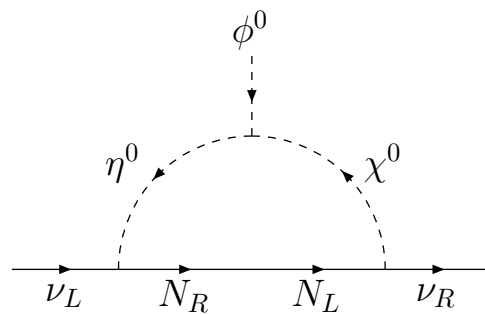
The usual theoretical thinking on neutrinos is that they should be Majorana. Given that there is still no experimental proof, i.e. no evidence of neutrinoless double beta decay, it is time that this idea is re-examined. The usual argument goes like this. For  $\nu_L$  to acquire mass,  $\nu_R$  should be added to the SM, but then  $\nu_R$  is allowed to have a large Majorana mass, hence  $\nu_L$  gets a small seesaw mass and everyone is happy. However,  $\nu_R$  is a trivial singlet in the SM and its existence is not required.

To enforce its existence, the SM should be extended, including gauge  $B - L$  for example. In that case, the breaking of  $B - L$  by two units would allow  $\nu_R$  to have a Majorana mass as usual, but breaking it by three units would not. This means that a residual global  $U(1)$  remains which protects the neutrino as Dirac fermion [8]. Depending on the details of the new particle content, the new lepton symmetry may be  $Z_3$  [9] or  $Z_4$  [10] or  $Z_n$  ( $n \geq 5$ ).

Combining this recent insight with that on DM, new models of Dirac neutrinos and dark matter are possible. Using gauge  $B - L$ , instead of having three  $\nu_R \sim 1$ , the theory is also anomaly-free with three right-handed neutral singlet fermions transforming as  $4, 4, -5$  [11]. In that case, tree-level Dirac neutrino masses are forbidden, but they may be generated radiatively by adding a suitable set of new fermions and scalars. Three recent studies are Refs. [12, 13, 14].

#### 5. SCOTOGENIC DIRAC NEUTRINO MASS WITH $Z_N^L$ AND $Z_N^D$ ( $N \geq 5$ )

To obtain a radiative Dirac neutrino mass induced by dark matter (scotogenic), three symmetries are usually assumed [15, 16]: (A) conventional lepton number, where  $\nu_{L,R}, N_{L,R}$  have  $L = 1$ , and  $\Phi, \eta, \chi$  have  $L = 0$ , which is strictly conserved; (B) dark  $Z_2$  symmetry, under which  $N_{L,R}, \eta, \chi$  are odd and others are even, which is strictly conserved; and (C) an *ad hoc*  $Z_2$  symmetry under which  $\nu_R, \chi$  are odd and all others even, which is softly broken by  $\eta^\dagger \Phi \chi$ .



**FIGURE 3:** Scotogenic Dirac neutrino mass.

To obtain exactly the same one-loop diagram, it has been shown recently [17] that a softly broken  $U(1)_L$  by itself will do the job. Consider the following particle content, as shown in Table 1.

**TABLE 1:**Fermion and scalar content for scotogenic Dirac neutrino mass.

fermion/scalar	$SU(2)$	$U(1)_Y$	$U(1)_L$	*	$Z_n^L$	$Z_n^D$
$(\nu, e)_L$	2	-1/2	1	1	$\omega$	1
$e_R$	1	-1	1	1	$\omega$	1
$\nu_R$	1	0	$x$	$-n+1$	$\omega$	1
$N_L$	1	0	$y$	$2-n$	$\omega^2$	$\omega$
$n_R$	1	0	$y$	$2-n$	$\omega^2$	$\omega$
$\Phi = (\phi^+, \phi^0)$	2	1/2	0	0	1	1
$\eta = (\eta^+, \eta^0)$	2	1/2	$y-1$	$1-n$	$\omega$	$\omega$
$\chi^0$	1	0	$y-x$	1	$\omega$	$\omega$

Here  $x \neq 1$  is imposed so that  $\nu_R$  does not couple to  $\nu_L$  at tree level. To connect them in one loop, the trilinear  $\bar{\eta}^0 \phi^0 \chi^0$  term must break  $U(1)_L$  softly by  $x-1$ . The  $y$  charge of  $N_{L,R}$  must not be  $\pm 1$  or  $\pm x$  to avoid undesirable couplings to  $\nu_{L,R}$ . The soft terms  $N_L N_L$  or  $N_R N_R$  would break  $U(1)_L$  by  $2y$ ,  $\nu_R \nu_R$  by  $2x$ ,  $N_R \nu_R$  by  $x+y$ ,  $\bar{N}_L \nu_R$  by  $x-y$ , and  $\chi^0 \chi^0$  by  $2(y-x)$ . They should be absent, hence they must not be zero or divisible by  $x-1$ . The column denoted by \* shows a class of solutions where  $U(1)_L$  breaks to  $Z_n$ , i.e.  $x = -n+1$  and  $y = 2-n$ .

If  $n = 3$ , then  $x+y = -3$ . If  $n = 4$ , then  $2y = -4$ . Hence  $n = 3, 4$  are ruled out. Any  $n \geq 5$  works. This results in two related symmetries: (I)  $Z_n^L$  lepton symmetry under which  $\nu_{L,R}, e_{L,R}, \eta, \chi \sim \omega$  and  $N_{L,R} \sim \omega^2$ , where  $\omega^n = 1$ ; (II)  $Z_n^D$  dark symmetry, derivable from  $Z_n^L$  by multiplying it by  $\omega^{-2j}$  where  $j$  is the particle's spin. As a result,  $\nu_{L,R}, e_{L,R} \sim 1$  and  $N_{L,R}, \eta, \chi \sim \omega$ . This is the Dirac generalization of  $\pi_D = (-1)^{2j} \pi_L$  for Majorana neutrinos.

In a renormalizable theory, the  $Z_n$  symmetry is not simply realizable. For  $n \geq 5$ ,  $(\chi^0)^n$  is not admissible. Hence the Lagrangian actually has a redefined  $U(1)_L$  symmetry under which  $\nu_{L,R}, e_{L,R}, \eta, \chi \sim 1$  and  $N_{L,R} \sim 2$ . The dark symmetry is then  $U(1)_D$  where it is derived from  $U(1)_L$  by subtracting  $2j$ , i.e.  $\nu_{L,R}, e_{L,R} \sim 0$  and  $N_{L,R}, \eta, \chi \sim 1$ .

If  $Z_n$  symmetry is desired, the scalar sector must be expanded. If  $n = 5$ , let  $\sigma \sim 3$  and  $\kappa \sim 7$  be added. Then the terms  $\chi^3 \sigma^*$ ,  $\chi^2 \sigma$ ,  $\chi \sigma^2 \kappa^*$ , and  $\kappa N_R \nu_R$  are allowed. Together they would enforce  $Z_5^L$  and  $Z_5^D$ .

A possible variation is to add  $\zeta \sim n$  and require  $U(1)_L$  to be spontaneously broken in the  $\zeta^* \eta^+ \Phi \chi$  term, thereby yielding a massless Goldstone boson, i.e. the diracon [18], as the analog of the majoron for Majorana neutrinos. A further application is to allow  $\zeta$  to couple anomalously to exotic color fermion triplets or a color fermion octet [19]. The diracon becomes the QCD axion and  $U(1)_L$  is extended Peccei-Quinn symmetry, as proposed long ago [20] for Majorana neutrinos, and very recently for Dirac neutrinos [21, 22].

## 6. SCOTOGENIC DIRAC NEUTRINO MASS WITH $Z_3^D$

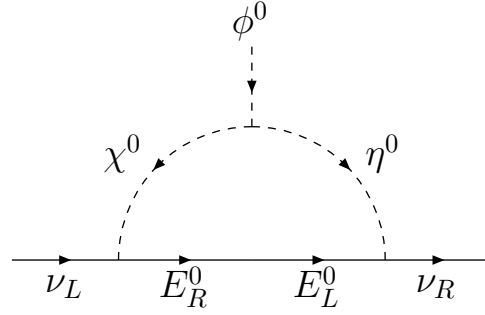
The  $N_{L,R}$  fermion singlets may be replaced by  $(E^0, E^-)_{L,R}$  fermion doublets, as shown in Table 2.

**TABLE 2:**Fermion and scalar content for scotogenic  $Z_3$  Dirac neutrino mass.

fermion/scalar	$SU(2)$	$U(1)_Y$	$U(1)_L$	**	$L$	$Z_3^D$
$(\nu, e)_L$	2	-1/2	1	1	1	1
$e_R$	1	-1	1	1	1	1
$\nu_R$	1	0	$x$	-2	1	1
$(E^0, E^-)_L$	2	-1/2	$y$	2	1	$\omega$
$(E^0, E^-)_R$	2	-1/2	$y$	2	1	$\omega$
$\Phi = (\phi^+, \phi^0)$	2	1/2	0	0	0	1
$\eta = (\eta^+, \eta^0)$	2	1/2	$x-y$	-4	0	$\omega^{-1}$
$\chi^0$	1	0	$y-1$	1	0	$\omega$

This construction eliminates the existence of many fermion bilinears except  $\nu_R \nu_R$  and  $\bar{\nu}_L E_R^0 + e_L^+ E_R^-$ . Hence only  $2x$  and  $y-1$  must not be zero or divisible by  $x-1$ . Also  $y \neq x$  is required. Now  $Z_3^D$  is possible as shown in the column denoted by \*\*. Here  $U(1)_L$  is broken by the soft  $\Phi^+ \eta \chi$  and  $\chi^3$  terms. However the dimension-four term  $\chi^0 \nu_R \nu_R$  is not allowed by the original  $U(1)_L$  even though it is allowed by  $Z_3^D$ . Hence the usual lepton assignment holds:  $L = 1$  for  $\nu_{L,R}, e_{L,R}$  and  $L = 0$  for all scalars. [Note that in  $Z_3$ , if  $\nu_R \sim \omega$ , then  $\chi \sim \omega$  or  $\omega^2$ . Hence either  $\chi \nu_R \nu_R$  or  $\chi^* \nu_R \nu_R$  must exist and  $\chi$  cannot be stable.]

In this example,  $U(1)_L$  is anomalous. To make it anomaly-free, the three copies of  $\nu_R$  with charge  $-2$  should be changed to 1. The difference is then  $3[1 - (-2)] = 3[3] = 9$  for the sum of the charges, and  $3[1 - (-8)] = 3[9] = 27$  for the sum of the cubes of the charges. This may be accomplished with singlet right-handed fermions  $\psi_{2,3,4}$  with charges  $-2, 3, -4$ . For 9 copies of  $\psi_3$  and 3 copies of  $\psi_{2,4}$ ,  $3[3(3) + (-2) + (-4)] = 3[3] = 9$  and  $3[3(27) + (-8) + (-64)] = 3[9] = 27$ . Add  $\zeta_{3,6}$  to break  $U(1)_L$  to  $Z_3^D$ . Then  $\psi_{2,3,4}$  have  $L = 1, 0, -1$ .



**FIGURE 4:** Scotogenic Dirac neutrino mass.

## 7. CONCLUDING REMARKS

The notion of generalized  $U(1)_L$  is useful in connecting leptons to dark matter. If it breaks softly to  $Z_2$ , then many DM prototype models may be understood in terms of lepton parity  $\pi_L$  (conserved for Majorana neutrinos) alone, with dark parity  $\pi_D = (-1)^{2j} \pi_L$ .

In scotogenic models,  $\pi_L$  and  $U(1)_D$  are possible together. For Dirac neutrinos, softly broken  $U(1)_L$  may also lead to  $Z_n^L$  and  $Z_n^D$  with  $n \geq 5$ , or redefined  $U(1)_L$  and  $U(1)_D$ . An example of  $Z_3^D$  and conventional  $L$  is also possible.

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