Tri-bimaximal-Cabibbo Mixing: flavour violation in charged lepton sector

Mathew Thomas Arun,[1] and Anirudhan A. Madathil,[1]

[1] *School of Physics, Indian Institute of Science Education and Research Thiruvananthapuram, Vithura, Kerala, 695551, India*

Abstract

The well understood structure of U_{pmns} matrix mandates a Cabibbo mixing matrix in the first two generations of the charged lepton sector if we assume Tri-bimaximal mixing in the neutrino sector. This ansatz, called Tri-bimaximal-Cabibbo mixing, on the other hand, is ruled out immediately by charged lepton flavour violating decays of mesons. In this article, we aim to show that the resurrection of the theoretically well motivated Tri-bimaximal mixing scenario comes naturally within Minimal Flavour Violation hypothesis in the lepton sector. We analyse the flavour violating currents $\mu \to eee$, $\mu Ti \to eTi$, $\pi^0 \to e^+\mu^-$ and $K_L \to \mu^+e^-$ in this scenario and show that the New Physics that generates mixing among the charged lepton could lie within the reach of hadron colliders. Though the most stringent constrain on New Physics is $\geq \mathcal{O}(26\text{ TeV})$ for maximal coupling, more natural coupling relaxes it to $\geq \mathcal{O}(4\text{ TeV})$.

Keywords: Tri-bi-maximal mixing, flavour violation, charged lepton, minimal flavour violation
DOI: 10.31526/ACP.BSM-2023.19

1. INTRODUCTION

The discovery of neutrino oscillation [1] has been of fundamental importance in understanding the flavour mixing in the neutral lepton sector. One such mixing matrix, motivated from discrete family symmetry like the A_4, is the Tri-bimaximal mixing (U_{TB}) [2]. But a non-zero measurement of the reactor angle by RENO [3] and Daya Bay [4] experiments rule out this hypothesis. Nevertheless, since it is theoretically well motivated and the rest of the angles predicted match the experimental results well enough, various variants based on the Tri-bimaximal mixing have been postulated [5].

Tri-bimaximal-Cabibbo [5] (TBC) mixing is an ansatz that emerged from a Tri-bimaximal hypothesis by including the non-zero reactor angle in the mixing of charged lepton sector. But, mixing in the charged lepton sector will contribute to the lepton flavour violating processes like $\pi^0 \to e^+\mu^-, K_L \to \mu^+e^-, \mu N \to eN, \mu \to eee$ etc. Such decays are searched for at high-intensity experiments giving rise to stringent bounds of BR($\pi^0 \to e^+\mu^-) < 3.2 \times 10^{-10}$ [6] (NA62 CERN), BR($K_L \to \mu^+e^-) < 4.7 \times 10^{-12}$ [7] (BNL), BR($\mu Ti \to eTi) < 6.1 \times 10^{-13}$ [8] (SINDRUM II), BR($\mu \to eee) < 1 \times 10^{-12}$ [9] (SINDRUM). Using the most general set of operators, TBC mixing without any flavour symmetry in the lepton sector is then ruled out by the current limits on LFV decays $\pi^0 \to e^+\mu^-$ and $K_L \to \mu^+e^-$. We show that the Minimal Flavour Violation (MFV) hypothesis [10, 11], with a choice of basis including the Cabibbo mixing among the charged leptons, can protect TBC mixing ansatz from these strong bounds.

2. TBC MIXING INDUCED LFV

In general the mixing in the lepton sector (U_{pmns}) can be written as, $U_{pmns} = {U_e^L}^\dagger U_\nu^L$. Where U_e^L is the mixing in the charged lepton sector and U_ν^L is the mixing in the neutrino sector. If we assume the TBC ansatz then,

$$ {U_e^L}^\dagger = \begin{pmatrix} c_{12}^e & s_{12}^e e^{-i\delta_{12}^e} & 0 \\ -s_{12}^e e^{i\delta_{12}^e} & c_{12}^e & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \; U_\nu^L = U_{TB} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1} $$

where $c_{12}^e = \cos\theta_{12}^e$ and $s_{12}^e = \sin\theta_{12}^e$. Mixing in the first two generations of charged leptons gives a non-zero reactor angle in U_{pmns}, which satisfies the relation $\sin\theta_{13} = \sin\theta_{12}^e/\sqrt{2}$. The approximate relation $\theta_{13} \approx \theta_c/\sqrt{2}$ is obtained if we take $\theta_{12}^e \approx \theta_c \approx 13^0$. The atmospheric and solar angles, $\sin\theta_{12} = 1/\sqrt{3}$ and $\sin\theta_{23} = 1/\sqrt{2}$ are in agreement with the TBC ansatz. Since TBC ansatz satisfies the experimentally observed angles, it motivates the study of mixing in the charged lepton sector.

2.1. $K_L \to \mu^+e^-$

The Low Energy Effective Field theory operators need to be matched with operators in chiral perturbation theory to get meson decays. We use the matching and decay rate expressions used in Ref.[12, 13, 14]. We take the ratio of flavour conserving $\Gamma(K_L \to$

$\mu^+\mu^-$) and the flavour violating decays $\Gamma(K_L \to \mu^+ e^-)$ to cancel out the hadronic factors and is given as,

$$\frac{\Gamma(K_L \to \mu^+ e^-)}{\Gamma(K_L \to \mu^+ \mu^-)} = |(U_e^L)_{e\mu}|^2 \frac{|p_\mu|_{K_L \mu e}(m_{K_L}^2 - (m_\mu + m_e)^2)}{|p_\mu|_{K_L \mu\mu}(m_{K_L}^2 - (m_\mu + m_\mu)^2)} = 6.14 \times 10^{-2} \tag{2}$$

In the above equation $|(U_e^L)_{e\mu}|^2 \approx 0.05$. Using the experimental bounds on $\mathrm{BR}(K_L \to \mu^+\mu^-)_{exp} = (6.84 \pm 0.11) \times 10^{-9}$ [15] and $\mathrm{BR}(K_L \to \mu^+ e^-)_{exp} < 4.7 \times 10^{-12}$ [7] we get,

$$\frac{\Gamma(K_L \to \mu^+ e^-)_{exp}}{\Gamma(K_L \to \mu^+ \mu^-)_{exp}} < (0.687 \pm 0.011) \times 10^{-3} \tag{3}$$

This experimental result is in contradiction with the TBC mixing induced ratio given in Eq.(2), thus ruling out the TBC ansatz for operators containing down sector quarks. For completeness, we will also discuss the flavour violating pion decay.

3. MINIMAL FLAVOUR VIOLATION WITH TBC MIXING

The Minimal Flavour Violation (MFV) hypothesis assumes that the Standard Model (SM) Yukawa couplings are the only source of flavour symmetry breaking [10, 11]. This means all the higher dimensional operators should be constructed out of the SM Yukawa couplings, satisfying the flavour symmetry $\mathcal{G}_F : SU(3)_Q \times SU(3)_u \times SU(3)_d \times SU(3)_L \times SU(3)_e$. The Yukawa couplings are considered as non-dynamical fields (spurions) which transform under the flavour symmetry $\mathcal{G}_F : \mathcal{G}_{QF} \times \mathcal{G}_{LF}$ ($\mathcal{G}_{QF} : SU(3)_Q \times SU(3)_u \times SU(3)_d$, $\mathcal{G}_{LF} : SU(3)_L \times SU(3)_e$) as,

$$Y_u \sim (3, \bar{3}, 1), \quad Y_d \sim (3, 1, \bar{3}), \quad Y_e \sim (\bar{3}, 3).$$

Higher dimensional operators are constructed using these Yukawa couplings satisfying the flavour symmetry $\mathcal{G}_F$. For the purpose of this article, we will assume MFV in the lepton sector and not in the quark sector.

In the case of minimal field content, mass terms in the lepton sector are,

$$\mathcal{L} = -vY_e^{ij}\bar{e}_R^i e_L^j - \frac{v^2}{2\Lambda_{LN}} g_\nu^{ij} \bar{\nu}_L^{ci} \nu_L^j + h.c \tag{1}$$

where, Λ_{LN} is the energy scale of the lepton number violation and $v = 256$ GeV is the vacuum expectation value of the Higgs field.

3.1. Analysis

Various operators that we consider that satisfy the flavour symmetry are listed in Table 1. These operators in minimal field content

Scalar	Tensor	Vector
$\mathcal{O}^{S1} = (\bar{L}_L \Delta^\dagger Y_e^\dagger e_R)(\bar{d}_R \lambda^{S1} Q_L)$	$\mathcal{O}^{T1} = (\bar{L}_L \sigma^{\mu\nu} \Delta^\dagger Y_e^\dagger e_R)(\bar{d}_R \sigma_{\mu\nu} \lambda^{T1} Q_L)$	$\mathcal{O}^{V1} = (\bar{L}_L \gamma^\mu \Delta L_L)(\bar{Q}_L \lambda^{V1} \gamma_\mu Q_L)$
$\mathcal{O}^{S2} = -(\bar{L}_L \Delta^\dagger Y_e^\dagger e_R)(\bar{Q}_L \lambda^{S2} i\tau^2 u_R)$	$\mathcal{O}^{T2} = -(\bar{L}_L \sigma^{\mu\nu} \Delta^\dagger Y_e^\dagger e_R)(\bar{Q}_L \sigma_{\mu\nu} \lambda^{T2} i\tau^2 u_R)$	$\mathcal{O}^{V2} = (\bar{L}_L \gamma^\mu \Delta L_L)(\bar{L}_L \lambda^{V2} \gamma_\mu L_L)$
$\mathcal{O}^{S3} = (\bar{L}_L \Delta^\dagger Y_e^\dagger e_R)(\bar{e}_R \lambda^{S3} L_L)$		

Table 1: Operators satisfying flavour symmetry [11]. Q_L and L_L represents $SU(2)$ doublet quark and lepton. u_R, d_R and e_R represent $SU(2)$ singlet up quark, down quark and charged lepton respectively.

scenario with fermions in their mass basis are shown in Table 2

	MMFV operators in minimal field content scenario
$\mathcal{O}^{S1}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_R \lambda^{S1} u_L) + (\bar{e}_L U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_R \lambda^{S1} d_L)$
$\mathcal{O}^{T1}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L \sigma^{\mu\nu} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_R \sigma_{\mu\nu} \lambda^{T1} u_L) + (\bar{e}_L \sigma^{\mu\nu} U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_R \sigma_{\mu\nu} \lambda^{T1} d_L)$
$\mathcal{O}^{S2}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_L \lambda^{S2} u_R) - (\bar{e}_L U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{u}_L \lambda^{S2} u_R)$
$\mathcal{O}^{T2}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L \sigma^{\mu\nu} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{d}_L \sigma_{\mu\nu} \lambda^{T2} u_R) - (\bar{e}_L \sigma^{\mu\nu} U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{u}_L \sigma_{\mu\nu} \lambda^{T2} u_R)$
$\mathcal{O}^{S3}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{e}_R \lambda^{S3} \nu_L) + (\bar{e}_L U_{pmns} m_\nu^2 U_{pmns}^\dagger D_e e_R)(\bar{e}_R \lambda^{S3} e_L)$
$\mathcal{O}^{V1}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L \gamma^\mu m_\nu^2 \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu^2 U_{pmns}^\dagger e_L)(\bar{u}_L \lambda^{V1} \gamma_\mu u_L + \bar{d}_L \gamma_\mu \lambda^{V1} d_L)$
$\mathcal{O}^{V2}$	$\frac{\Lambda_{LFV}^2}{v^4}\,(\bar{\nu}_L \gamma^\mu m_\nu^2 \nu_L + \bar{e}_L \gamma^\mu U_{pmns} m_\nu^2 U_{pmns}^\dagger e_L)(\bar{\nu}_L \lambda^{V2} \gamma_\mu \nu_L + \bar{e}_L \lambda^{V2} \gamma_\mu e_L)$

Table 2: MMFV operators in minimal field content scenario.

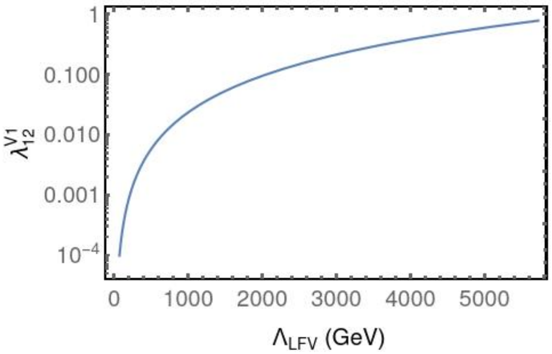

Figure 1: Limits on Λ_{LFV} from BR$(K_L \to \mu^+ e^-)_{exp}$ for different values of $\lambda_{12}^{V1}/\lambda_{12}^{S1}$ in vector operator scenarios with minimal field content scenario.

$K_L \to \mu^+ e^-$:
The operators that contribute to $K_L \to \mu^+ e^-$ are:

$$\begin{aligned} \mathcal{C}^{S1}_{e\mu ds}\mathcal{O}^{S1}_{e\mu ds} &= \Big(\bar{e}_L\,\{\Delta^\dagger Y_e^\dagger\}_{12}\,\mu_R\Big)(\bar{d}_R\,\lambda_{12}^{S1}\,s_L) \\ \mathcal{C}^{S1}_{e\mu sd}\mathcal{O}^{S1}_{e\mu sd} &= \Big(\bar{e}_L\,\{\Delta^\dagger Y_e^\dagger\}_{12}\,\mu_R\Big)(\bar{s}_R\,\lambda_{21}^{S1}\,d_L) \\ \mathcal{C}^{V1}_{e\mu ds}\mathcal{O}^{V1}_{e\mu ds} &= \Big(\bar{e}_L\,\gamma^\mu\Delta_{12}\,\mu_L\Big)(\bar{d}_L\,\gamma_\mu\lambda_{12}^{V1}\,s_L) \\ \mathcal{C}^{V1}_{e\mu sd}\mathcal{O}^{V1}_{e\mu sd} &= \Big(\bar{e}_L\,\gamma^\mu\Delta_{12}\,\mu_L\Big)(\bar{s}_L\,\gamma_\mu\lambda_{21}^{V1}\,d_L) \end{aligned} \tag{2}$$

We assume that $\lambda_{12}^{S1} = \lambda_{21}^{S1}$ and $\lambda_{12}^{V1} = \lambda_{21}^{V1}$. The branching ratio in case of scalar operator becomes,

$$\begin{aligned} \text{BR}(K_L \to \mu^+ e^-) &= \frac{|p_\mu|_{K_L\mu e}}{8\pi m^2_{K_L^0}}\frac{\tau_{K_L}B_0^2F_0^2}{4}\left|\left(\frac{\mathcal{C}^{S1}_{e\mu ds}}{\alpha}+\frac{\mathcal{C}^{S1}_{e\mu sd}}{\beta}\right)\right|^2(m^2_{K_L^0}-(m_\mu+m_e)^2) \\ \frac{|\mathcal{C}^{S1}_{e\mu ds}|^2}{\Lambda^4_{LFV}} &< 1.1\times10^{-25}\text{GeV}^{-4} \end{aligned} \tag{3}$$

$\tau_{K_L} = 5.116 \times 10^{-8}s$ is the mean life time of K_L. The branching ratio in case of vector operator is [14],

$$\begin{aligned} \text{BR}(K_L \to e^+ \mu^-) &= \frac{|p_\mu|_{K_L\mu e}}{8\pi m^2_{K_L^0}}\frac{\tau_{K_L}F_0^2}{4}\left|\left(\frac{\mathcal{C}^{V1}_{e\mu ds}}{\Lambda^2_{LFV}\alpha}+\frac{\mathcal{C}^{V1}_{e\mu sd}}{\Lambda^2_{LFV}\beta}\right)\right|^2(m^2_{K_L^0}(m^2_\mu+m^2_e)-(m^2_\mu-m^2_e)^2) \\ \frac{|\mathcal{C}^{V1}_{e\mu ds}|^2}{\Lambda^4_{LFV}} &< 7.04\times10^{-23}\text{GeV}^{-4} \end{aligned} \tag{4}$$

The limit on Λ_{LFV} from BR$(K_L \to \mu^+ e^-)_{exp}$ in vector operator scenario for different values of λ is shown in Fig. 1. In the case of scalar operator, only the extended field scenario gives significant constrain on Λ_{LFV}.
$\mu N \to eN$:
The operators that contribute to $\mu N \to eN$ are:

$$\begin{aligned} \mathcal{C}^{S1}_{e\mu uu}\mathcal{O}^{S1}_{e\mu uu} &= \Big(\bar{e}_L\,\{\Delta^\dagger Y_e^\dagger\}_{12}\,\mu_R\Big)(\bar{d}_R\,\lambda_{11}^{S1}\,d_L) \\ \mathcal{C}^{S2}_{e\mu dd}\mathcal{O}^{S2}_{e\mu dd} &= \Big(\bar{e}_L\,\{\Delta^\dagger Y_e^\dagger\}_{12}\,\mu_R\Big)(\bar{u}_R\,\lambda_{11}^{S2}\,u_L) \\ \mathcal{C}^{V1}_{e\mu qq}\mathcal{O}^{V1}_{e\mu qq} &= \Big(\bar{e}_L\,\gamma^\mu\Delta_{12}\,\mu_L\Big)(\bar{u}_L\,\gamma_\mu\lambda_{11}^{V1}\,u_L+\bar{d}_L\,\gamma_\mu\lambda_{11}^{V1}\,d_L) \end{aligned} \tag{5}$$

The best limit on $\mu \to e$ conversion comes from the SINDRUM II collaboration, which puts a limit on BR$(\mu Ti \to eTi)_{exp} < 6.1 \times 10^{-13}$ [8].

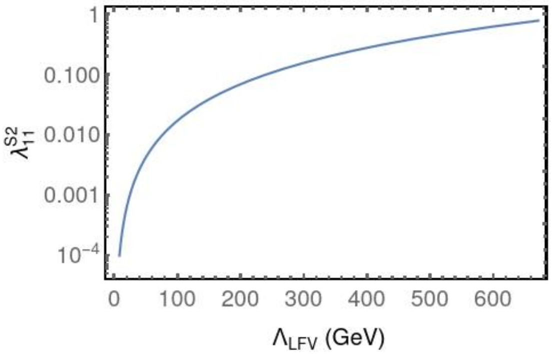

Figure 2: Limits on Λ_{LFV} from BR($\mu Ti \to eTi$) for different values of $\lambda_{11}^{S2}/\lambda_{11}^{V1}$ in scalar operators in minimal field content scenarios.

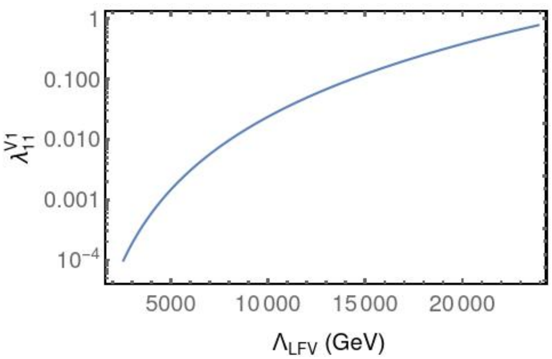

Figure 3: Limits on Λ_{LFV} from BR($\mu Ti \to eTi$) for different values of $\lambda_{11}^{S2}/\lambda_{11}^{V1}$ in vector operators in minimal field content scenarios.

For scalar operator we consider a scenario when $\lambda^{S1} = 0$ and $\lambda^{S2} \neq 0$, in this case $\mathcal{C}_{e\mu dd}^{S2} = \frac{G_F}{\sqrt{2}} g_{LS(d)}$,

$$\text{BR}(\mu Ti \to eTi) = \frac{4}{w_{capt}} (G_S^{(d,p)} S^{(p)} + G_S^{(d,n)} S^{(n)})^2 \frac{|\mathcal{C}_{e\mu dd}^{S2}|^2}{\Lambda_{LFV}^4}$$

$$\frac{|\mathcal{C}_{e\mu dd}^{S2}|^2}{\Lambda_{LFV}^4} < 1.37 \times 10^{-25} \text{GeV}^{-4} \quad (6)$$

In the case of a vector operator $\frac{G_F}{\sqrt{2}} g_{LV(d)} = \frac{G_F}{\sqrt{2}} g_{LV(u)} = \mathcal{C}_{e\mu dd}^{V1} = \mathcal{C}_{e\mu uu}^{V1}$,

$$\text{BR}(\mu Ti \to eTi) = \frac{36}{w_{capt}} (V^{(p)} + V^{(n)})^2 \frac{|\mathcal{C}_{e\mu dd}^{V1}|^2}{\Lambda_{LFV}^4}$$

$$\frac{|\mathcal{C}_{e\mu dd}^{V1}|^2}{\Lambda_{LFV}^4} < 2.94 \times 10^{-25} \text{GeV}^{-4} \quad (7)$$

Fig. 2 and Fig. 3 shows the limit on Λ_{LFV} from BR$(\mu Ti \to eTi)_{exp}$ for different values of $\lambda_{11}^{S2}/\lambda_{11}^{V1}$ in scalar and vector operator scenarios. We see that among all the lepton flavour violating decays considered, the highest limit on Λ_{LFV} is from the vector operator scenario of BR($\mu Ti \to eTi$). The vector operator constraints Λ_{LFV} more compared to the scalar operator. This is expected since vector operators are of the order m_ν^2, were as the scalar operator is of the order $m_\nu^2 m_e$.

4. CONCLUSION

TBC mixing ansatz gives a possible venue for NP that could contribute towards charged LFV. This scenario also brings back the thought to be dead, but theoretically well-motivated, Tri-bimaximal mixing in the neutrino sector. In this paper, we investigate the LFV induced by TBC mixing due to mixing in charged lepton sector in the context of various lepton flavour violating decays like $\mu \to eee$, $\pi^0 \to \mu e$, $K_L \to \mu e$ and $\mu N \to eN$. A model-independent analysis using the effective field theory operators revealed that if we assume TBC mixing to be the only source of LFV then it can be ruled out by current limits on lepton flavour violating decays $K_L \to \mu^+ e^-$ and $\pi^0 \to e^+ \mu^-$.

Whereas, as we discuss, a natural symmetry like the MFV hypothesis can protect TBC mixing from these lepton flavour violating decays, rendering the NP available at LHC. Moreover, the neutrino mixing matrix can still be Tri-bimaximal. In Table 3 we summarise the limit on Λ_{LFV} for the minimal field content scenario in the lepton sector. The strongest constrain on Λ_{LFV} comes from BR($\mu Ti \to eTi$). Assuming $\mathcal{O}(1)$ couplings in the quark sector, this process bounds the cut-off scale to be $\Lambda_{LFV} \geq 26$ TeV. Instead, if we assume a slightly lower coupling of NP in the quark sector, this limit could be relaxed to $\Lambda_{LFV} \geq 2$ TeV.

Thus, the lepton flavour violating NP is not only closer to the scale of experiments, but also the neutrino mixing matrix is safe from the reactor angle. This also means that a theoretically sound neutrino sector exists only with the MFV hypothesis.

Observables	Scenario	Limit on Λ_{LFV} (TeV)	Scenario	Limit on Λ_{LFV} (TeV)
BR($\pi^0 \to e^+\mu^-$)	$\lambda_{11}^{S1} = 1$ $\lambda_{11}^{V1} = 1$	4×10^{-5} 6.5×10^{-3}	$\lambda_{11}^{S1} = \frac{m_d}{v}$ $\lambda_{11}^{V1} = \frac{m_d}{v}$	2×10^{-7} 3.4×10^{-5}
BR($\mu^- \to e^-e^+e^-$)	$\lambda_{11}^{S3} = 1$ $\lambda_{11}^{V2} = 1$	5.558×10^{-2} 3.82	$\lambda_{11}^{S3} = c_{12}^e$ $\lambda_{11}^{V2} = c_{12}^e$	5.158×10^{-2} 3.75
BR ($K_L \to \mu^+e^-$)	$\lambda_{12}^{S1} = 1$ $\lambda_{12}^{V1} = 1$	0.793 6.427	$\lambda_{12}^{S1} = \frac{m_s}{v}$ $\lambda_{12}^{V1} = \frac{m_s}{v}$	1.184×10^{-2} 0.14
BR($\mu Ti \to eTi$)	$\lambda_{11}^{S2} = 1$ $\lambda_{11}^{V1} = 1$	0.752 25.287	$\lambda_{11}^{S2} = \frac{m_d}{v}$ $\lambda_{11}^{V1} = \frac{m_d}{v}$	3.9×10^{-3} 1.82

Table 3: Limit on Λ_{LFV} from different lepton flavour violating decays in minimal field content scenario.

ACKNOWLEDGEMENT

M.T.A. acknowledges the financial support of DST through INSPIRE Faculty grant DST/INSPIRE/04/2019/002507.

References

[1] Y. Fukuda et al. Evidence for oscillation of atmospheric neutrinos. *Phys. Rev. Lett.*, 81:1562–1567, 1998.
[2] P. F. Harrison, D. H. Perkins, and W. G. Scott. Tri-bimaximal mixing and the neutrino oscillation data. *Phys. Lett. B*, 530:167, 2002.
[3] J. K. Ahn et al. Observation of Reactor Electron Antineutrino Disappearance in the RENO Experiment. *Phys. Rev. Lett.*, 108:191802, 2012.
[4] F. P. An et al. Observation of electron-antineutrino disappearance at Daya Bay. *Phys. Rev. Lett.*, 108:171803, 2012.
[5] S. F. King. Tri-bimaximal-Cabibbo Mixing. *Phys. Lett. B*, 718:136–142, 2012.
[6] Eduardo Cortina Gil et al. Search for Lepton Number and Flavor Violation in K^+ and π^0 Decays. *Phys. Rev. Lett.*, 127(13):131802, 2021.
[7] D. Ambrose et al. New limit on muon and electron lepton number violation from K0(L) —> mu+- e-+ decay. *Phys. Rev. Lett.*, 81:5734–5737, 1998.
[8] J. Kaulard et al. Improved limit on the branching ratio of mu- –> e+ conversion on titanium. *Phys. Lett. B*, 422:334–338, 1998.
[9] U. Bellgardt et al. Search for the Decay mu+ —> e+ e+ e-. *Nucl. Phys. B*, 299:1–6, 1988.
[10] G. D'Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia. Minimal flavor violation: An Effective field theory approach. *Nucl. Phys. B*, 645:155–187, 2002.
[11] Vincenzo Cirigliano, Benjamin Grinstein, Gino Isidori, and Mark B. Wise. Minimal flavor violation in the lepton sector. *Nucl. Phys. B*, 728:121–134, 2005.
[12] J. Gasser and H. Leutwyler. Chiral Perturbation Theory to One Loop. *Annals Phys.*, 158:142, 1984.
[13] J. Gasser and H. Leutwyler. Chiral Perturbation Theory: Expansions in the Mass of the Strange Quark. *Nucl. Phys. B*, 250:465–516, 1985.
[14] Arun Mathew Thomas, Priyanka Lamba, and Sudhir K. Vempati. Restricting q^2l^2 operators from $\pi^0 \to \mu e$. 4 2022.
[15] R. L. Workman. Review of Particle Physics. *PTEP*, 2022:083C01, 2022.