Constraints on the Exotic Spin-dependent Interactions from Magnetometers Data

L.Y. Wu,[1,2,3], H. Yan,[2,3]

[1]*Key Laboratory of Nuclear Physics and Ion-beam Application (MOE), Fudan University, Shanghai 200433, China*

[2]*Key Institute of Nuclear Physics and Chemistry, CAEP, Mianyang, Sichuan 621900, China*

[3]*Key Laboratory of Neutron Physics, Institute of Nuclear Physics and Chemistry, CAEP, Mianyang 621900, Sichuan, China*

Abstract

The existence of exotic spin-dependent interactions is predicted by various new physics theories. These interactions generate macroscopic forces on polarized spins that can be detected with a magnetometer. The Sun and Moon could be considered mass sources capable of generating such interactions through axion, Z' boson, or unparticle exchange. New experimental upper limits on exotic spin-dependent interactions mediated by axions or Z' bosons at astronomical ranges are derived by analyzing magnetometer data measuring Lorentz and CPT violation. Additionally, the first upper limits on monopole-dipole spin-dependent interactions mediated by unparticles are obtained.

Keywords: Exotic spin-dependent interactions, Atomic magnetometer, beyond the Standard Model

DOI: 10.31526/ACP.BSM-2023.18

1. INTRODUCTION

In the Standard Model (SM), the spin dominates its interactions with the environment through the electroweak and strong forces. However other possible interactions of spin with the environment are not precluded. These hypothetical exotic spin-dependent interactions could have wide-ranging implications for fundamental physics, such as new sources of CP violation [1], tests of the weak equivalence principle and Lorentz or CPT invariance [2, 3], and hunting for dark matter [4]. In recent years, an important driving force for the search for exotic spin-dependent interactions has come from the search for new particles [5]. Many theories beyond the SM predict the existence of new particles, such as the QCD axion [6, 7, 8], axion-like-particles (ALPs) [9], and Z' boson [10]. One common specialty of these particles is that they might mediate spin-dependent interactions between fermions [5]. These new light bosons are also prominent candidates for dark matter [11, 12]. Beyond the quest for new particles, unparticles are another motivation for the search for exotic spin-dependent interactions. It is an attempt to incorporate the scale invariance into the SM [13]. The coupling of unparticles to fermions will result in spin-dependent interactions that depend on a nonintegral power of the distance between the fermions [14, 15].

Given the importance of exotic spin-dependent interactions in the search for new physics, many approaches have been adopted to search for them, such as torsion balance [16, 17, 18], cantilever [19, 20], maser [21], nitrogen-vacancy (NV) center in diamond [22, 23], nuclear magnetometer resonance [24, 25], and atomic magnetometer [26, 27, 28, 29]. Especially, atomic magnetometers, due to their ultrasensitivity to magnetic fields, have been widely used in the search for spin-dependent interactions [30]. Searching for the spin-dependent interactions is typically conducted by positioning a bulk object around the magnetometers, and measuring the effective magnetic field generated by the object. For example, an optical polarized ^{3}He is utilized as a spin source and a spin-based amplifier based on ^{87}Rb-^{129}Xe comagnetometer is installed aside to sense the spin-spin-dependent interactions at a typical interaction range of centimeters [31]. Using the Earth as an unpolarized mass source, Zhang et al. search the exotic spin-dependent interaction at the range of 10^5 km using a ^{129}Xe-^{131}Xe comagnetometer [32]. For a new particle with a mass of m, the typical range of the force it mediates is parameterized by the Compton wavelength $\lambda = \hbar c/m$, where c is the speed of light in vacuum. Most laboratory experiments focus on searching for exotic spin-dependent interactions with a range smaller than the scale of the Earth's radius [5], corresponding to a new particle mass greater than 10^{-14} eV. However, exotic spin-dependent interactions over astronomical distances, which could be mediated by much lighter particles, have rarely been studied in laboratory experiments. This letter provides a concise overview of our two recent works, as detailed in [33, 34]. In Ref. [33], we used the Sun and Moon as mass sources and combined the old magnetometers data to constrain the exotic spin-dependent interactions through axion and Z' exchange at astronomical distances. In Ref. [34], we derived the monopole-dipole spin-dependent interactions between fermions mediated by unparticle. And we used magnetometers data to establish some limits on these interactions.

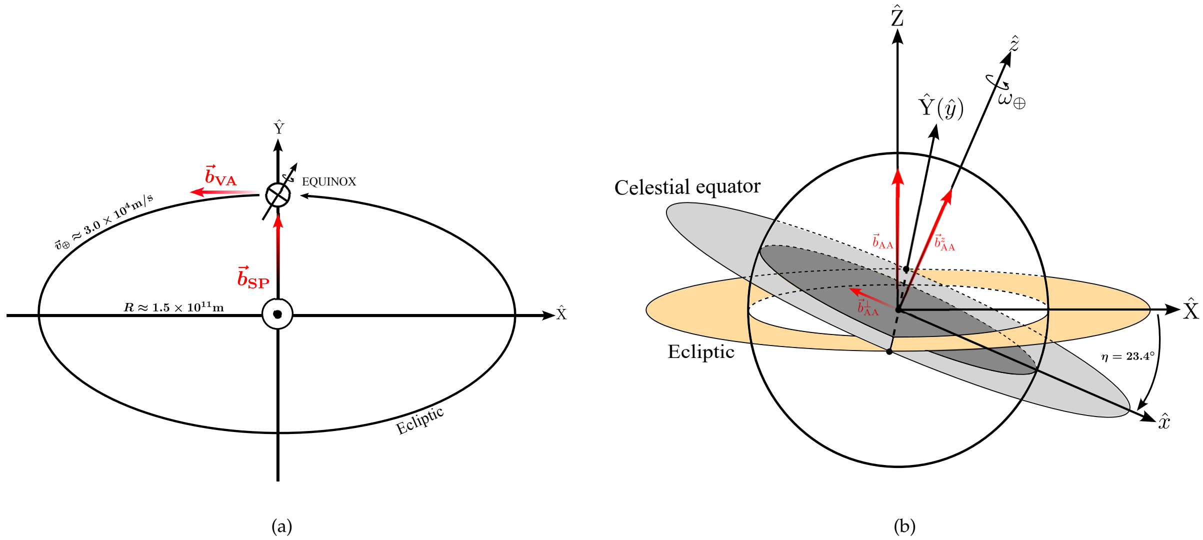

FIGURE 1: (a) The Sun-centered frame. The relative size of the Sun and the Earth is not to scale. (b) The Earth-based frame. We take $\hat{z}$ along with the Earth's rotation axis. The angle between the ecliptic plane and the Earth's equatorial plane is $\eta = 23.4°$. The red arrows represent the directions of effective fields of three types (SP, VA, AA) of spin-dependent interactions generated by the Sun's nucleons. Figure adopted from Ref. [33].

2. EXOTIC SPIN-DEPENDENT INTERACTIONS

The QCD axion and ALPs have similar properties of interacting with fermions, except that the QCD axion has a definite relationship between its mass and interaction strength, while ALPs do not. In experiments, the techniques of searching for QCD axion and ALPs are similar. Therefore, the exotic spin-dependent interactions search is generally considered in the ALPs model that covers broader parameter space. In this context, we will use the term "axion" to refer to the QCD axion and ALPs. We focus on the interactions between fermion ψ and axion ϕ, and the massive Z' boson. The coupling of the axion to a Dirac fermion ψ has the general form of [9]

$$\mathcal{L}_{a\bar{\psi}\psi} = g_S \phi \bar{\psi}\psi + \frac{g_P}{2m_\phi}\partial^\mu \phi \bar{\psi}\gamma_\mu \gamma_5 \psi, \tag{1}$$

where g_S and g_P are respectively the dimensionless scalar and pseudoscalar coupling strength, and m_ϕ is the mass of the axion. In the non-relativistic limit, the monopole-dipole interaction through one axion exchange can be written as

$$V_{\mathrm{SP}} = \frac{g_S g_P}{8\pi m_\psi}\left(\frac{1}{\lambda_\phi r} + \frac{1}{r^2}\right)\exp(-r/\lambda_\phi)\boldsymbol{\sigma}\cdot\hat{r}, \tag{2}$$

where m_ψ is the mass of the fermion, $\boldsymbol{\sigma}$ is the spin of the particle, $\hat{r}$ is the unit vector, and λ_ϕ is the interaction range. Here we adopt the natural units ($\hbar = c = 1$). The mass range of axion is extensive. The allowed mass range for the QCD axion is 10^{-13} to 10^{-2} eV, while for ALPs, the mass can extend down to the Hubble scale of 10^{-33} eV [5, 9]. Thus, the axion might mediate interactions in ranges from nanometers to astronomical distances. This interaction violates the party (P) and time-reversal (T) symmetry, thus not CP conserving [35]. In atomic phenomena, this potential will contribute to the electric-dipole-moments (EDMs) in atoms or molecules. The measurement of the EDMs provides a test of such interaction at atomic scale [36].

The Z' boson raised from the $U(1)'$ symmetry interacts with the neutral current of the fermions as follows [10]

$$\mathcal{L}_{Z'\bar{\psi}\psi} = Z'_\mu \bar{\psi}(g_V \gamma^\mu + g_A \gamma_5 \gamma^\mu)\psi, \tag{3}$$

where g_V and g_A are respectively the dimensionless vector and axial-vector coupling strength. The exchange of Z' boson between fermions will induce the spin-velocity-dependent interactions

$$\begin{aligned} V_{\mathrm{VA}} &= \frac{g_V g_A}{2\pi}\frac{\exp(-r/\lambda_{Z'})}{r}\boldsymbol{\sigma}\cdot\vec{v}, \\ V_{\mathrm{AA}} &= \frac{g_A g_A}{16\pi m_\psi}\left(\frac{1}{\lambda_{Z'} r} + \frac{1}{r^2}\right)\exp(-r/\lambda_{Z'})\boldsymbol{\sigma}\cdot(\vec{v}\times\hat{r}), \end{aligned} \tag{4}$$

where $\vec{v}$ is the relative velocity between two fermions, and $\lambda_{Z'}$ is inversely proportional to the mass of the Z' boson. Here we only enumerate these three kinds of monopole-dipole spin-dependent interactions related to our discussion, a more comprehensive review of all the possible interactions could be referred from [37, 38].

In addition to spin-dependent interactions arising from the exchange of new particles, the coupling of the SM to a hidden sector with scale invariance also manifests spin-dependent interactions. In fact, unlike our familiar SM fields, the quantum excitation of the scale-invariant sector could not be described in terms of particle, therefore dubbed unparticle. The unparticle does not enjoy mass as one of its intrinsic properties and is not even constrained by the dispersion relation. The kinematic property of an unparticle is determined by its scaling dimension $d_{\mathcal{U}}$ [13, 39]. The leading order interaction of the unparticle with fermions in the effective field theory is [14]

$$\mathcal{L}_{\mathcal{U}\bar{\psi}\psi} = \Phi_{\mathcal{U}}\bar{\psi}(C_S + i\gamma_5 C_P)\psi + X^{\mu}_{\mathcal{U}}\bar{\psi}(C_V\gamma_\mu + \gamma_\mu\gamma_5 C_A)\psi, \tag{5}$$

where $\Phi_{\mathcal{U}}$ and $X^{\mu}_{\mathcal{U}}$ stand for the scalar and vector unparticles, and C_S, C_P, C_V, C_A are the coupling constants with the energy dimension of $1 - d_{\mathcal{U}}$. These coupling constants could be parameterized by $C_i = c_i\Lambda_{\mathcal{U}}^{1-d_{\mathcal{U}}}$ with c_i a dimensionless constant and $\Lambda_{\mathcal{U}}$ an unknown energy scale. Generally, $\Lambda_{\mathcal{U}}$ is fixed to 1 TeV, and considers the constraint on c_i. Similarly, in the non-relativistic limit, we could derive three types of spin-dependent interactions

$$\begin{aligned}
U_{\mathrm{SP}}(r) &= -C_S C_P \frac{A_{d_{\mathcal{U}}}}{8\pi^2}\frac{\Gamma(2d_{\mathcal{U}})}{(2d_{\mathcal{U}}-2)m_1 r^{2d_{\mathcal{U}}}}\boldsymbol{\sigma}\cdot\hat{r},\\
U_{VA}(r) &= C_V C_A \frac{A_{d_{\mathcal{U}}}}{4\pi^2}\frac{1}{r^{2d_{\mathcal{U}}-1}}\Gamma(2d_{\mathcal{U}}-2)\boldsymbol{\sigma}\cdot\vec{v},\\
U_{AA}(r) &= C_A^2 \frac{A_{d_{\mathcal{U}}}}{16m\pi^2}\frac{\Gamma(2d_{\mathcal{U}})}{2d_{\mathcal{U}}-2}\frac{1}{r^{2d_{\mathcal{U}}}}\boldsymbol{\sigma}\cdot(\vec{v}\times\hat{r}),
\end{aligned} \tag{6}$$

where $\Gamma(x)$ is the gamma function, and $A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}}\frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}$ is the normalization factor. We can observe that these interactions adhere to a power-law dependency, resembling a characteristic of electromagnetic forces. The performance of these interactions is dictated by the scaling factor $d_{\mathcal{U}}$. Conventionally, the range of $d_{\mathcal{U}}$ takes $(1, 2)$. Previously, only the dipole-dipole spin-dependent interactions were widely considered [14, 15, 40, 41]. In our recent work [34], we extended the interactions to monopole-dipole interactions, and six kinds of spin-dependent interactions are derived, including scalar-scalar (SS), scalar-pseudoscalar (SP), pseudoscalar-pseudoscalar (PP), vector-vector (VV), vector-axial-vector (VA), and axial-axial-vector (AA) types. Eq. (6) is part of these six interactions. A more comprehensive expression of all the potentials derived from Lagrangian (5) could be found in Ref. [34].

3. DETECTION OF THE INTERACTIONS

Regardless of the different origins of all these spin-dependent interactions, they could be recast as

$$V = -\hbar\gamma\boldsymbol{\sigma}\cdot\boldsymbol{B}_{\mathrm{eff}}, \tag{1}$$

where γ is the gyromagnetic ratio, and $\boldsymbol{B}_{\mathrm{eff}}$ is the effective magnetic field produced by these interactions. The induced effective magnetic field results in an energy shift between opposite spin states, akin to the Zeeman effect caused by an actual magnetic field. Searching for these exotic interactions now becomes a problem of searching for the effective magnetic field acting on the spins. As aforementioned, these spin-dependent interactions could produce effects over macroscopic distances. Therefore, the minute effective magnetic field produced by every particle may coherently superimpose to yield a substantial collective influence at the scale of λ. Experimentally, a mass source or a highly polarized spin source is usually applied as a source of the effective magnetic field.

The atomic magnetometer is a sensitive probe that uses polarized spin to detect the magnetic field acting on it. Its working principle is as follows. The magnetic field exerts a torque, $\boldsymbol{\mu}\times\boldsymbol{B}$, on spins possessing a magnetic moment $\boldsymbol{\mu}$. Consequently, these spins undergo Larmor precession at a frequency proportional to the field's magnitude. The frequency is read out to determine the magnetic field imposed on the spins. According to different spin species utilized in the magnetometers, it can be divided into electron spin magnetometer, nuclear spin magnetometer, and comagnetometer with mixed spin species. Notably, comagnetometers exhibit inherent magnetic field suppression, retaining sensitivity to fields induced by exotic spin-dependent interactions. Consequently, they find extensive application in this domain [42]. On the one hand, the sensitivity of these magnetometers to the exotic interactions is determined by their magnetic resolution limit. The state-of-art magnetometer could reach a magnetic resolution of about sub fT/$\sqrt{\mathrm{Hz}}$ [43]. The fundamental limit of the resolution is decided by the quantum noise, such as spin projection noise and photon scattering noise, therefore further improvement on the sensitivity may be enhanced by spin squeezing [44, 45]. On the other hand, the mundane effect caused by the ambient magnetic field should also be carefully considered. Since $\boldsymbol{B}_{\mathrm{eff}}$ is not mediated by photons, it does not obey Maxwell's equations. Hence, certain magnetic shielding techniques could be employed to diminish the influence of the magnetic field without attenuating the effective magnetic field.

4. EXPERIMENTAL CONSTRAINTS

4.1. Constraints on axion and Z' boson

The Sun is a giant mass source containing about $N_\odot \approx 1.2 \times 10^{57}$ unpolarized nucleons. A polarized neutron spin on the Earth is assumed to interact with the nucleons of the Sun through spin-dependent interaction V_{SP} (1). Since there is a relative velocity between the Sun and Earth, the spin-velocity-dependent interactions V_{VA} and V_{AA} (4) mediated by Z' could also be studied. The direction of these three interactions is shown in Fig. 1. In the frame of the Earth's celestial coordinates, The components of these interactions perpendicular to the Earth's rotation axis are sidereal modulated. The corresponding oscillating effective magnetic field $\boldsymbol{b}_\perp$ are

$$\begin{aligned}
\boldsymbol{b}_{\text{SP}\perp} =& \frac{\hbar g_S^N g_P^n N_\odot}{4\pi m_n \gamma_n}\left(\frac{1}{\lambda_\phi R} + \frac{1}{R^2}\right)\exp\left(-R/\lambda_\phi\right) \\
& [\sin\left(\omega_\oplus t\right)\hat{x} + \cos\left(\omega_\oplus t\right)\hat{y}], \\
\boldsymbol{b}_{\text{VA}\perp} =& -\frac{g_V^N g_A^n N_\odot}{\pi\gamma_n}\frac{\exp(-R/\lambda_{Z'})}{R} v_\oplus \cos\eta \\
& [-\cos\left(\omega_\oplus t\right)\hat{x} + \sin\left(\omega_\oplus t\right)\hat{y}], \\
\boldsymbol{b}_{\text{AA}\perp} =& \frac{\hbar g_A^N g_A^n N_\odot}{8\pi m_n c \gamma_n}\left(\frac{1}{\lambda_{Z'} R} + \frac{1}{R^2}\right)\exp(-R/\lambda_{Z'}) v_\oplus \sin\eta \\
& [\cos\left(\omega_\oplus t\right)\hat{x} - \sin\left(\omega_\oplus t\right)\hat{y}],
\end{aligned} \tag{1}$$

where γ_n is the gyromagnetic ratio of the neutron, R the average distance between the Sun and Earth, $\omega_\oplus$ the frequency of the Earth's rotation, η the Earth's obliquity, and $v_\oplus = \omega_\oplus R$ the orbital speed of Earth. g_i^N represents the coupling between axion or Z' boson with the Sun's nucleons, and g_i^n represents the coupling between axion or Z' boson with the polarized neutron, and $i = S, P, V, A$.

The sidereal modulation due to the Earth's rotation is also applied in the search of a background field caused by Lorentz violation [48, 49]. The Lorentz violation will induce a constant background field $\tilde{\boldsymbol{b}}$ spreading in the universe, this field interacts with the spin by $\tilde{\boldsymbol{b}} \cdot \boldsymbol{\sigma}$. In Ref [48, 49], a ^{3}He-^{129}Xe comagnetometer and a K-^{3}He one was adopted, respectively, to search for this background field. Both are terrestrial experiments, therefore the background field sensed by the comagnetometer is sidereal modulated. We found that the Lorentz violation background field searching results could be applied to constrain the spin-dependent interactions we are interested in.
Using the result of Ref. [48], we derived the effective field of the three types of spin-dependent interactions generated by the Sun's nucleons as

$$|b_\perp| \leq 0.023 \text{ fT}. \tag{2}$$

All the parameters in Eq. (1) are known, we could derive the constraints to the coupling constants $|g_S^N g_P^n|$, $|g_V^N g_A^n|$, and $|g_A^N g_A^n|$ as a function of the interaction range λ. Using the Moon as a mass source, we could obtain the constraints using the same analyzing method. Our obtained constraints are shown in Figs. 2-4. The dark gray region of the coupling strength is excluded. The most stringent constraints are reached when the interaction range is $\sim 10^{12}$ m for the Sun and $\sim 10^9$ m for the Moon. This is understandable since when the interaction range $\lambda \gg R$, these interactions are independent of λ. And when $\lambda < R$, the interactions are suppressed

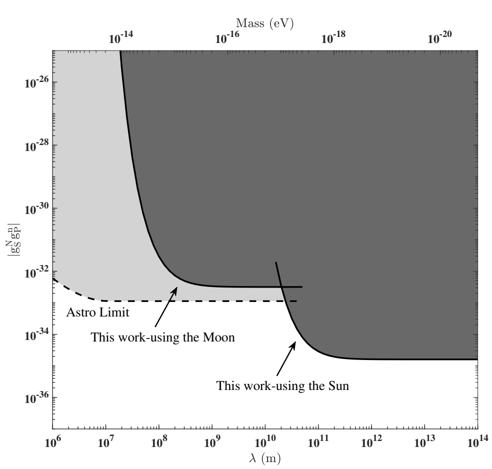

FIGURE 2: Constraint to the coupling constant $|g_S^N g_P^n|$ versus interaction range λ. Solid lines show constraints calculated in our recent work [33] using the Moon (left line) and Sun (right line) as mass sources. The dashed line is the result of Refs. [46, 47], which was derived by combining g_S^N of weak equivalence with g_P^n from SN1987A. The dark gray area is excluded by our result and the light gray one is excluded by the result of Refs. [46, 47]. Figure adopted from Ref. [33].

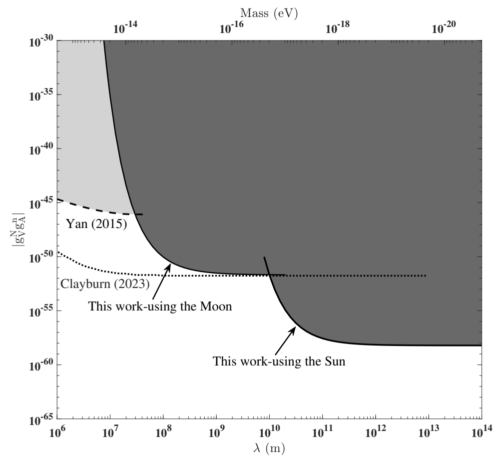

FIGURE 3: Constraint to the coupling constant $|g_V^N g_A^n|$ versus interaction range λ. Solid lines show constraints calculated in our recent work [33] using the Moon (left line) and Sun (right line) as mass sources, and the dark grey area is excluded. The dashed line is the result of Ref. [50] and the light gray area is excluded. The dotted line is the result of Ref. [51]. Figure adopted from Ref. [33].

by the exponential factor, thereby less stringent constraints are yielded.

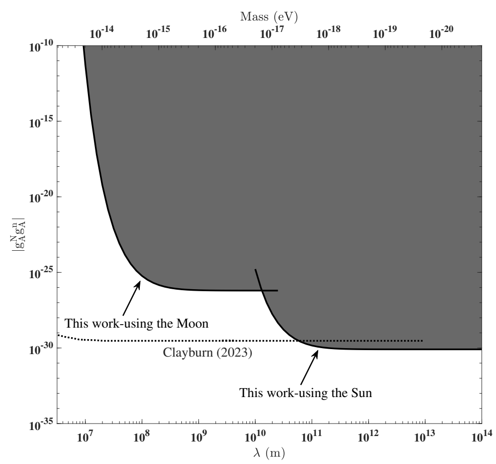

FIGURE 4: Constraint to the coupling constant $|g_A^N g_A^n|$ versus interaction range λ. Solid lines show constraints calculated in our recent work [33] using the Moon (left line) and Sun (right line) as mass sources. The dark grey area is excluded. The dotted line is the result of Ref. [51]. Figure adopted from Ref. [33].

In Fig. 2, the dashed line and the light gray region is the result of a combination of laboratory (g_S^N) and astrophysical (g_P^n) bounds. For $\lambda > 10^{12}$ m, our bounds $|g_S^N g_P^n| < 1.6 \times 10^{-35}$ (95% CL). This result improves the combined astrophysical-laboratory bound by about 70 times. Figure 3 shows the constraint on $|g_V^N g_A^n|$. The dashed line and dotted line are the results of Refs. [50, 51]. Comparing with the newest published result of Ref. [51], our bounds $|g_V^N g_A^n| < 7.1 \times 10^{-59}$ (95% CL) are improved by about 6 orders of magnitude. The constraint on $|g_A^N g_A^n|$ is shown in Fig. 4. For $\lambda > 10^{12}$ m, our bounds $|g_A^N g_A^n| < 8.1 \times 10^{-31}$ (95% CL). The dotted line is the result of Ref. [51].

4.2. Constraints on unparticle

Assuming the spin-dependent interactions between the nucleon in the Sun and the polarized neutron are mediated by unparticle, we could derive the constraints of the interactions listed in Eq. (6). The constraints on $c_S^N c_P^n$, $c_V^N c_A^n$, and $c_A^N c_A^n$ are shown as the blue solid line in Figs. 5-7.

The $\mathcal{U}_{SP}(r)$ potential, originating from the coupling of different vertex types (scalar and pseudoscalar vertex), violates the CP symmetry. The CP violating interactions could contribute to the nonzero EDMs in atoms or molecules. Based on the precise measurement of the EDM of ^{199}Hg atom [53], we obtain a constraint on $c_S c_P$ between electron and nucleon (e-N) at $d_{\mathcal{U}} = 1.0$, shown as the green triangle in Fig. 5. The constraint between N-e marked in orange diamond is derived from the result of Ref. [52], a

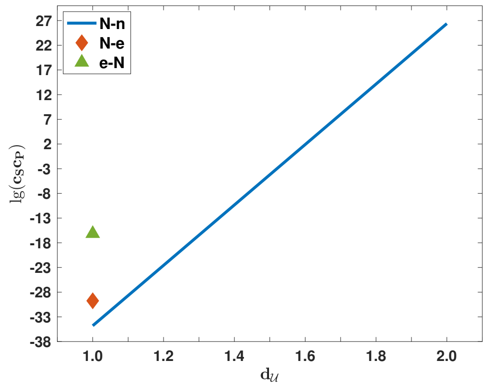

FIGURE 5: Constraints on the N-e and e-N SP interactions ($c_S^N c_P^e$ and $c_S^e c_P^N$) derived respectively from the results of Refs. [52, 53] and those on the N-n SP interaction ($c_S^N c_P^n$) derived from the results of Ref. [33]. Figure adopted from Ref. [34].

magnetometer-based experiment that searches for exotic spin-dependent interactions. For the $\mathcal{U}_{AA}(r)$ potential, we also set constraints on $c_A c_A$ between electron and neutron (e-n) and electron-electron (e-e) at $d_{\mathcal{U}} = 1.0$ using the results of [54, 55], and the results are marked in green triangle and orange diamond in Fig. 7. The constraints on e-e marked in stars in Fig. 7 are the results of Ref. [14], where experimental data on exotic spin-spin dependent interactions search are analyzed.

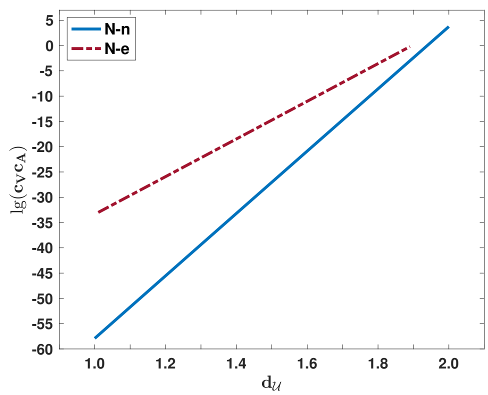

FIGURE 6: Constraints on the N-n and N-e VA interactions ($c_V^N c_A^n$ and $c_V^N c_A^e$) derived from the results of Refs. [33, 28]. Figure adopted from Ref. [34].

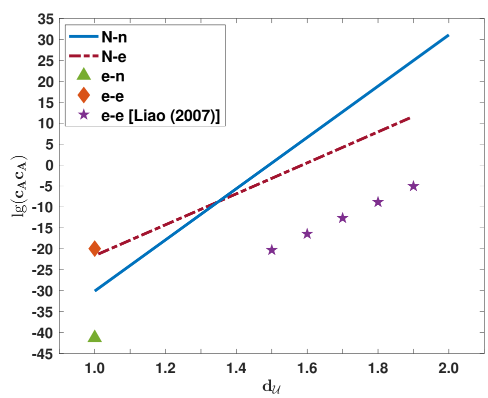

FIGURE 7: Constraints on the e-e, e-n, N-n, and N-e AA interactions derived from the results of Refs. [54, 55, 33, 28], in comparison with those on the e-e interaction in Ref. [14]. Figure adopted from Ref. [34].

The red dot-dashed lines in Fig. 6 and 7 are derived from a magnetometer-based experiment [28] that searches for the interactions in Eq. (4). In Ref. [28], two cylindrical BGO crystals were used as sources of the effective magnetic field, and the crystals were rotated by a servo motor. Four Spin-Exchange-Relaxation-Free (SERF) magnetometers were arranged surrounding the BGO crystals to detect the exotic spin-dependent interaction between the nucleon in the crystal and the polarized electrons in the magnetometer. Here if the spin-dependent interactions were mediated by unparticle, we could derive the effective magnetic field for different values of $d_{\mathcal{U}}$. The effective magnetic field generated by the BGO crystals are

$$\begin{aligned}\mathbf{B}_{\mathrm{VA}}(r) &= \frac{2(\hbar c)^{2d_{\mathcal{U}}-2}}{\gamma_e} c_V^N c_A^e \Lambda^{2-2d_{\mathcal{U}}} \frac{A_{d_{\mathcal{U}}}}{4\pi^2} \Gamma(2d_{\mathcal{U}}-2) \\ &\int_V \frac{1}{|\boldsymbol{r}-\boldsymbol{r'}|^{2d_{\mathcal{U}}-1}} \boldsymbol{v}(\boldsymbol{r'}) d\boldsymbol{r'}^3, \\ \mathbf{B}_{\mathrm{AA}}(r) &= \frac{2(\hbar c)^{2d_{\mathcal{U}}-1}}{c^2\gamma_e m_N} c_A^N c_A^e \Lambda^{2-2d_{\mathcal{U}}} \frac{A_{d_{\mathcal{U}}}}{4\pi^2} \frac{\Gamma(2d_{\mathcal{U}})}{2d_{\mathcal{U}}-2} \\ &\int_V \frac{1}{|\boldsymbol{r}-\boldsymbol{r'}|^{2d_{\mathcal{U}}}} \boldsymbol{v}(\boldsymbol{r'}) \times \frac{\boldsymbol{r}-\boldsymbol{r'}}{|\boldsymbol{r}-\boldsymbol{r'}|} d\boldsymbol{r'}^3,\end{aligned} \tag{3}$$

where the integral is over the volume of the BGO crystal, $\boldsymbol{r'}$ $(\boldsymbol{v}(\boldsymbol{r'}))$ denotes the position (velocity) of the volume element of the BGO crystal, $\boldsymbol{r}$ is the position of the magnetometer, and γ_e is the gyromagnetic ratio of the electron. This integral could be implemented through Monte Carlo integration when the coupling constant is assumed to be 1. The actual magnetic field detected by the magnetometers is compared with the simulated ones $\boldsymbol{B}_{\mathrm{VA}}$ and $\boldsymbol{B}_{\mathrm{AA}}$ to obtain the true coupling constants $c_V^N c_A^e$ and $c_A^N c_A^e$.
Note that the constraints on these three unparticle mediated interactions are linearly dependent on the value of $d_{\mathcal{U}}$ in the logarithmic scale, indicating these constraints exhibit an exponential decay trend as $d_{\mathcal{U}}$ increases. This relation is evident from the Eq. (3), which is an exponential function of $d_{\mathcal{U}}$.

5. CONCLUSION

We investigate the exotic spin-dependent interactions mediated by axion, Z' boson, and unparticles. The old magnetometer data from several experiments are used to set constraints on these interactions. For the interactions mediated by axion and Z' boson, we obtained upper bounds on the coupling constants $g_S^N g_P^n$, $g_V^N g_A^n$, and $g_A^N g_A^n$ at astronomical distances. Compared with the previous results, our results on $g_S^N g_P^n$ improve about 70 times and about 6 orders of magnitude on $g_V^N g_A^n$.
We derive the unparticle mediated monopole-dipole spin-dependent interactions, and constraints on them are obtained based on previous magnetometer data. The constraints exhibit an exponential decay trend as $d_{\mathcal{U}}$ increases. Since these interactions are exponentially dependent on the distance, the $d_{\mathcal{U}}$ determines the decay rate of these interactions as the distance increases. For large $d_{\mathcal{U}}$, the decay of the interactions caused by the distance may counteract the collective effect arising from the numerous particles, therefore yielding a loose constraint. On the contrary, in the atomic scale, these interactions are more significant for larger $d_{\mathcal{U}}$, which may imply better constraint for large $d_{\mathcal{U}}$ [56].

ACKNOWLEDGEMENTS

The authors thank the committee of the "Beyond Standard Model: From Theory to Experiment (BSM- 2023)" workshop. This work is supported by the National Natural Science Foundation of China under Grants No. U2230207, No. U2030209, No. 12305114 and No. 12305125. This work was also supported by the National Key Program for Research and Development of China under Grants No. 2020YFA0406001 and No. 2020YFA0406002.

References

[1] N. D. Hari Dass. Test for c, p, and t nonconservation in gravitation. *Phys. Rev. Lett.*, 36:393–395, Feb 1976.
[2] Wouter Dekens, Jordy de Vries, and Sachin Shain. Cp-violating axion interactions in effective field theory. *Journal of High Energy Physics*, 2022(7):14, July 2022.
[3] F. Allmendinger, W. Heil, S. Karpuk, W. Kilian, A. Scharth, U. Schmidt, A. Schnabel, Yu. Sobolev, and K. Tullney. New limit on lorentz-invariance- and c p t -violating neutron spin interactions using a free-spin-precession he 3 - xe 129 comagnetometer. *Physical Review Letters*, 112(11):110801, March 2014.
[4] Junyi Lee, Mariangela Lisanti, William A. Terrano, and Michael Romalis. Laboratory constraints on the neutron-spin coupling of fev-scale axions. *Physical Review X*, 13(1):011050, March 2023.
[5] M. S. Safronova, D. Budker, D. DeMille, Derek F. Jackson Kimball, A. Derevianko, and Charles W. Clark. Search for new physics with atoms and molecules. *Rev. Mod. Phys.*, 90:025008, Jun 2018.
[6] Luca Di Luzio, Maurizio Giannotti, Enrico Nardi, and Luca Visinelli. The landscape of QCD axion models. *Physics Reports*, 870:1–117, Jul 2020.
[7] R. D. Peccei and Helen R. Quinn. CP Conservation in the Presence of Pseudoparticles. *Physical Review Letters*, 38:1440–1443, Jun 1977.
[8] Peter Svrcek and Edward Witten. Axions in string theory. *Journal of High Energy Physics*, 2006(06):051–051, Jun 2006.
[9] Pierre Sikivie. Invisible axion search methods. *Reviews of Modern Physics*, 93(1):015004, February 2021.
[10] Paul Langacker. The physics of heavy Z' gauge bosons. *Rev. Mod. Phys.*, 81:1199–1228, Aug 2009.
[11] Gianfranco Bertone and Dan Hooper. History of dark matter. *Reviews of Modern Physics*, 90:045002, October 2018.

[12] Francesca Chadha-Day, John Ellis, and David J. E. Marsh. Axion dark matter: What is it and why now? *Science Advances*, 8(8):eabj3618, Feb 2022.

[13] Howard Georgi. Unparticle physics. *Phys. Rev. Lett.*, 98:221601, May 2007.

[14] Yi Liao and Ji-Yuan Liu. Long-range electron spin-spin interactions from unparticle exchange. *Phys. Rev. Lett.*, 99:191804, Nov 2007.

[15] N.G. Deshpande, Stephen D.H. Hsu, and Jing Jiang. Long range forces and limits on unparticle interactions. *Phys. Lett. B*, 659(5):888–890, 2008.

[16] E. G. Adelberger, B. R. Heckel, S. Hoedl, C. D. Hoyle, D. J. Kapner, and A. Upadhye. Particle-physics implications of a recent test of the gravitational inverse-square law. *Physical Review Letters*, 98(13):131104, March 2007.

[17] S. Schlamminger, K.-Y. Choi, T. A. Wagner, J. H. Gundlach, and E. G. Adelberger. Test of the equivalence principle using a rotating torsion balance. *Physical Review Letters*, 100(4):041101, January 2008.

[18] W. A. Terrano, E. G. Adelberger, J. G. Lee, and B. R. Heckel. Short-range, spin-dependent interactions of electrons: A probe for exotic pseudo-goldstone bosons. *Physical Review Letters*, 115(20):201801, November 2015.

[19] Jihua Ding, Jianbo Wang, Xue Zhou, Yu Liu, Ke Sun, Adekunle Olusola Adeyeye, Huixing Fu, Xiaofang Ren, Sumin Li, Pengshun Luo, Zhongwen Lan, Shanqing Yang, and Jun Luo. Constraints on the velocity and spin dependent exotic interaction at the micrometer range. *Physical Review Letters*, 124(16):161801, April 2020.

[20] Qian Wang, Ze Ouyang, Yu Lu, Jianbo Wang, Lin Zhu, and Pengshun Luo. Proposal for the search for exotic spin-spin interactions at the micrometer scale using functionalized cantilever force sensors. *Physical Review D*, 107(1):015005, January 2023.

[21] Alexander G. Glenday, Claire E. Cramer, David F. Phillips, and Ronald L. Walsworth. Limits on anomalous spin-spin couplings between neutrons. *Physical Review Letters*, 101(26):261801, December 2008.

[22] Xing Rong, Mengqi Wang, Jianpei Geng, Xi Qin, Maosen Guo, Man Jiao, Yijin Xie, Pengfei Wang, Pu Huang, Fazhan Shi, Yi-Fu Cai, Chongwen Zou, and Jiangfeng Du. Searching for an exotic spin-dependent interaction with a single electron-spin quantum sensor. *Nature Communications*, 9(1):739, December 2018.

[23] P.-H. Chu, N. Ristoff, J. Smits, N. Jackson, Y. J. Kim, I. Savukov, and V. M. Acosta. Proposal for the search for new spin interactions at the micrometer scale using diamond quantum sensors. *Physical Review Research*, 4(2):023162, May 2022.

[24] Asimina Arvanitaki and Andrew A. Geraci. Resonantly detecting axion-mediated forces with nuclear magnetic resonance. *Physical Review Letters*, 113(16):161801, October 2014.

[25] Antoine Garcon, Deniz Aybas, John W Blanchard, Gary Centers, Nataniel L Figueroa, Peter W Graham, Derek F Jackson Kimball, Surjeet Rajendran, Marina Gil Sendra, Alexander O Sushkov, Lutz Trahms, Tao Wang, Arne Wickenbrock, Teng Wu, and Dmitry Budker. The cosmic axion spin precession experiment (casper): a dark-matter search with nuclear magnetic resonance. *Quantum Science and Technology*, 3(1):014008, dec 2017.

[26] Young Jin Kim, Ping-Han Chu, Igor Savukov, and Shaun Newman. Experimental limit on an exotic parity-odd spin- and velocity-dependent interaction using an optically polarized vapor. *Nature Communications*, 10(1):2245, May 2019.

[27] Kai Wei, Wei Ji, Changbo Fu, Arne Wickenbrock, Victor V. Flambaum, Jiancheng Fang, and Dmitry Budker. Constraints on exotic spin-velocity-dependent interactions. *Nature Communications*, 13(1):7387, November 2022.

[28] K. Y. Wu, S. Y. Chen, G. A. Sun, S. M. Peng, M. Peng, and H. Yan. Experimental limits on exotic spin and velocity dependent interactions using rotationally modulated source masses and an atomic-magnetometer array. *Phys. Rev. Lett.*, 129:051802, Jul 2022.

[29] Kai Wei, Tian Zhao, Xiujie Fang, Zitong Xu, Chang Liu, Qian Cao, Arne Wickenbrock, Yanhui Hu, Wei Ji, Jiancheng Fang, and Dmitry Budker. Ultrasensitive atomic comagnetometer with enhanced nuclear spin coherence. *Physical Review Letters*, 130(6):063201, February 2023.

[30] Derek F. Jackson Kimball, Dmitry Budker, Timothy E. Chupp, Andrew A. Geraci, Shimon Kolkowitz, Jaideep T. Singh, and Alexander O. Sushkov. Probing fundamental physics with spin-based quantum sensors. *Phys. Rev. A*, 108:010101, Jul 2023.

[31] Yuanhong Wang, Haowen Su, Min Jiang, Ying Huang, Yushu Qin, Chang Guo, Zehao Wang, Dongdong Hu, Wei Ji, Pavel Fadeev, Xinhua Peng, and Dmitry Budker. Limits on axions and axionlike particles within the axion window using a spin-based amplifier. *Physical Review Letters*, 129(5):051801, July 2022.

[32] Shaobo Zhang, Zengli Ba, Denghui Ning, Nianfu Zhai, Zhengtian Lu, and Dong Sheng. Search for spin-dependent gravitational interactions at the earth range, March 2023.

[33] L. Y. Wu, K. Y. Zhang, M. Peng, J. Gong, and H. Yan. New limits on exotic spin-dependent interactions at astronomical distances. *Phys. Rev. Lett.*, 131:091002, Aug 2023.

[34] L. Y. Wu, K. Y. Zhang, and H. Yan. Exotic spin-dependent interactions through unparticle exchange, 2023.

[35] Ciaran A. J. O'Hare and Edoardo Vitagliano. Cornering the axion with c p -violating interactions. *Physical Review D*, 102(11):115026, December 2020.

[36] V. A. Dzuba, V. V. Flambaum, I. B. Samsonov, and Y. V. Stadnik. New constraints on axion-mediated p,t-violating interaction from electric dipole moments of diamagnetic atoms. *Phys. Rev. D*, 98:035048, Aug 2018.

[37] Bogdan A Dobrescu and Irina Mocioiu. Spin-dependent macroscopic forces from new particle exchange. *Journal of High Energy Physics*, 2006(11):005–005, November 2006.

[38] Pavel Fadeev, Yevgeny V. Stadnik, Filip Ficek, Mikhail G. Kozlov, Victor V. Flambaum, and Dmitry Budker. Revisiting spin-dependent forces mediated by new bosons: Potentials in the coordinate-space representation for macroscopic- and atomic-scale experiments. *Physical Review A*, 99(2):022113, February 2019.

[39] Howard Georgi. Another odd thing about unparticle physics. *Phys. Lett. B*, 650(4):275–278, 2007.

[40] Michael F. Wondrak, Piero Nicolini, and Marcus Bleicher. Unparticle contribution to the hydrogen atom ground state energy. *Phys. Lett. B*, 759:589–592, 2016.

[41] Tanmay Kumar Poddar. Constraints on ultralight axions, vector gauge bosons, and unparticles from geodetic and frame-dragging measurements. *Eur. Phys. J. C*, 82(11):982, 2022.

[42] W A Terrano and M V Romalis. Comagnetometer probes of dark matter and new physics. *Quantum Science and Technology*, 7(1):014001, nov 2021.

[43] I. K. Kominis, T. W. Kornack, J. C. Allred, and M. V. Romalis. A subfemtotesla multichannel atomic magnetometer. *Nature*, 422(6932):596–599, April 2003.

[44] Florian Wolfgramm, Alessandro Cerè, Federica A. Beduini, Ana Predojević, Marco Koschorreck, and Morgan W. Mitchell. Squeezed-light optical magnetometry. *Physical Review Letters*, 105(5):053601, July 2010.

[45] R. J. Sewell, M. Koschorreck, M. Napolitano, B. Dubost, N. Behbood, and M. W. Mitchell. Magnetic sensitivity beyond the projection noise limit by spin squeezing. *Physical Review Letters*, 109(25):253605, December 2012.

[46] Georg Raffelt. Limits on a cp-violating scalar axion-nucleon interaction. *Phys. Rev. D*, 86:015001, Jul 2012.

[47] Ciaran A. J. O'Hare and Edoardo Vitagliano. Cornering the axion with cp-violating interactions. *Phys. Rev. D*, 102:115026, Dec 2020.

[48] F. Allmendinger, W. Heil, S. Karpuk, W. Kilian, A. Scharth, U. Schmidt, A. Schnabel, Yu. Sobolev, and K. Tullney. New limit on lorentz-invariance- and *cpt*-violating neutron spin interactions using a free-spin-precession ^{3}He-^{129}Xe comagnetometer. *Phys. Rev. Lett.*, 112:110801, Mar 2014.

[49] J. M. Brown, S. J. Smullin, T. W. Kornack, and M. V. Romalis. New limit on lorentz- and *cpt*-violating neutron spin interactions. *Phys. Rev. Lett.*, 105:151604, Oct 2010.

[50] H. Yan, G. A. Sun, S. M. Peng, Y. Zhang, C. Fu, H. Guo, and B. Q. Liu. Searching for new spin- and velocity-dependent interactions by spin relaxation of polarized ^{3}He gas. *Phys. Rev. Lett.*, 115:182001, Oct 2015.

[51] N. B. Clayburn and L. R. Hunter. Using earth to search for long-range spin-velocity interactions. *Phys. Rev. D*, 108:L051701, Sep 2023.

[52] Junyi Lee, Attaallah Almasi, and Michael Romalis. Improved limits on spin-mass interactions. *Phys. Rev. Lett.*, 120:161801, Apr 2018.

[53] B. Graner, Y. Chen, E. G. Lindahl, and B. R. Heckel. Reduced limit on the permanent electric dipole moment of ^{199}Hg. *Phys. Rev. Lett.*, 116:161601, Apr 2016.

[54] Attaallah Almasi, Junyi Lee, Himawan Winarto, Marc Smiciklas, and Michael V. Romalis. New limits on anomalous spin-spin interactions. *Phys. Rev. Lett.*, 125:201802, Nov 2020.

[55] Wei Ji, Yao Chen, Changbo Fu, Ming Ding, Jiancheng Fang, Zhigang Xiao, Kai Wei, and Haiyang Yan. New experimental limits on exotic spin-spin-velocity-dependent interactions by using smco$_5$ spin sources. *Phys. Rev. Lett.*, 121:261803, Dec 2018.

[56] Michael F. Wondrak, Piero Nicolini, and Marcus Bleicher. Unparticle contribution to the hydrogen atom ground state energy. *Phys. Lett. B*, 759:589–592, 2016.