Interpreting the Cosmic History of the Universe Through $\mathcal{N}=2$ $\mathcal{D}=5$ Supergravity

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Abstract

Through modeling the universe as a symplectic 3-brane embedded in the bulk of $\mathcal{N}=2$ five-dimensional ungauged supergravity theory, the entire evolution of the universe can be interpreted from inflation to late-time acceleration without introducing an inflaton nor a cosmological constant.

Keywords: Supergravity, Dark energy, Inflation, Cosmological evolution, Extra dimensions
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1. INTRODUCTION

We aim to explain the entire cosmic history of the universe during its different acceleration phases from inflation till the present accelerated expansion [1, 2] based on topological effects and the existence of an extra dimension. We model the universe as a symplectic 3-brane embedded in the bulk of $\mathcal{N}=2$ $\mathcal{D}=5$ supergravity that has been produced from the dimensional reduction of $\mathcal{D}=11$ $\mathcal{N}=1$ supergravity over Calabi-Yau 3-fold [3]. The brane is filled by dust matter and radiation, whilst its cosmological constant vanishes. To fit the recent observations of the ΛCDM model [4], the bulk should be a de Sitter space with a tiny positive cosmological constant $(0.02 < \tilde{\Lambda} < 0.03)$ [Gyr^{-2}].

2. FIVE DIMENSIONAL $\mathcal{N}=2$ SUPERGRAVITY

The ungauged five dimensional $\mathcal{N}=2$ supergravity theory contains two sets of matter fields; the vector multiplets, which we set to zero, and our main interest: the *hypermultiplets*. These are composed of the *universal hypermultiplet* $\left(\phi,\sigma,\zeta^0,\tilde{\zeta}_0\right)$; where ϕ is the universal axion, and the dilaton σ is proportional to the volume of the underlying Calabi-Yau manifold $\mathcal{M}$. The remaining hypermultiplet scalars are $\left(z^i, z^{\bar{i}}, \zeta^i, \tilde{\zeta}_i : i=1,\ldots,h_{2,1}\right)$, where the z's are the complex structure moduli of $\mathcal{M}$, and $h_{2,1}$ is the Hodge number determining the dimensions of the manifold $\mathcal{M}_C$ of the Calabi-Yau's complex structure moduli. The fields $\left(\zeta^I, \tilde{\zeta}_I : I=0,\ldots,h_{2,1}\right)$ are the axions, which define a symplectic vector space (see [5] for a review and more references). The axions are defined as components of the symplectic vector

$$|\Xi\rangle = \begin{pmatrix} \zeta^I \\ -\tilde{\zeta}_I \end{pmatrix}, \tag{1}$$

The bosonic part of the action is given by:

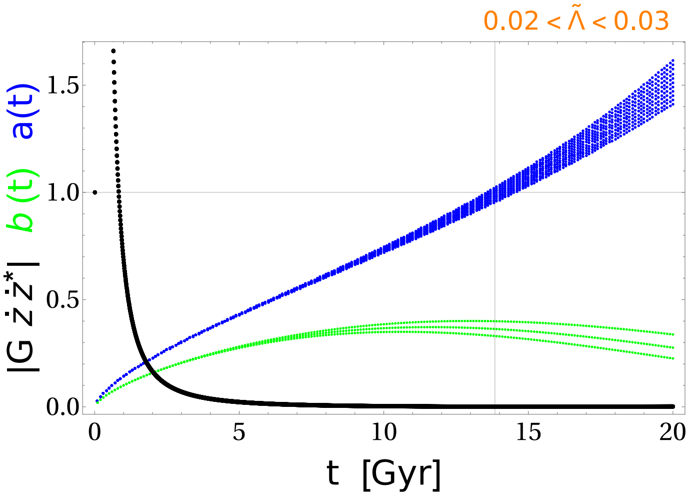
$$\begin{aligned} S_5 \quad = \quad \int_5 \Big[& \mathcal{R}\star\mathbf{1} - \frac{1}{2}d\sigma\wedge\star d\sigma - G_{i\bar{j}}dz^i\wedge\star dz^{\bar{j}} + e^{\sigma}\langle d\Xi|\underset{\wedge}{\mathbf{\Lambda}}|\star d\Xi\rangle \\ & -\frac{1}{2}e^{2\sigma}\left[d\phi+\langle\Xi\mid d\Xi\rangle\right]\wedge\star\left[d\phi+\langle\Xi\mid d\Xi\rangle\right]\Big]. \end{aligned} \tag{2}$$

Where $\mathcal{R}$ is the curvature scalar of the five-dimensional metric g_{MN}, $M,N=0,...,4$. The bulk's stress tensor is:

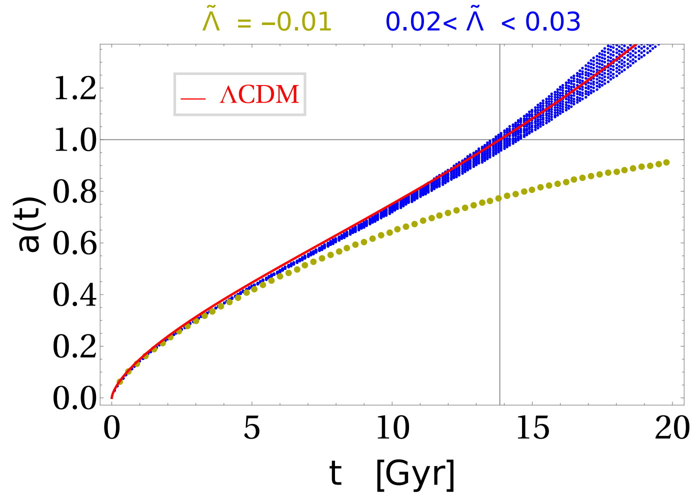
$$\begin{aligned} T_{\mu\nu} \quad = \quad & -\frac{1}{2}\left(\partial_\mu\sigma\right)\left(\partial_\nu\sigma\right)+\frac{1}{4}g_{\mu\nu}\left(\partial_\alpha\sigma\right)\left(\partial^\alpha\sigma\right)+e^{\sigma}\left\langle\partial_\mu\Xi\right|\mathbf{\Lambda}\left|\partial_\nu\Xi\right\rangle-\frac{1}{2}e^{\sigma}g_{\mu\nu}\left\langle\partial_\alpha\Xi\right|\mathbf{\Lambda}\left|\partial^\alpha\Xi\right\rangle \\ & -\frac{1}{2}e^{2\sigma}\left[\left(\partial_\mu\phi\right)+\left\langle\Xi\mid\partial_\mu\Xi\right\rangle\right]\left[\left(\partial_\nu\phi\right)+\left\langle\Xi\mid\partial_\nu\Xi\right\rangle\right]+\frac{1}{4}e^{2\sigma}g_{\mu\nu}\left[\left(\partial_\alpha\phi\right)+\left\langle\Xi\mid\partial_\alpha\Xi\right\rangle\right]\left[\left(\partial^\alpha\phi\right)+\left\langle\Xi\mid\partial^\alpha\Xi\right\rangle\right] \\ & -G_{i\bar{j}}\left(\partial_\mu z^i\right)\left(\partial_\nu z^{\bar{j}}\right)+\frac{1}{2}g_{\mu\nu}G_{i\bar{j}}\left(\partial_\alpha z^i\right)\left(\partial^\alpha z^{\bar{j}}\right). \end{aligned} \tag{3}$$

Where $\mu,\nu=0,...,3$. As our main interest is bosonic configurations that preserve *some* supersymmetry, the stress tensor can be simplified by considering the vanishing of the supersymmetric variations [6]; satisfying the BPS condition on the brane. This gives

$$T_{\mu\nu} = G_{i\bar{j}}\left(\partial_\mu z^i\right)\left(\partial_\nu z^{\bar{j}}\right)-\frac{1}{2}g_{\mu\nu}G_{i\bar{j}}\left(\partial_\alpha z^i\right)\left(\partial^\alpha z^{\bar{j}}\right). \tag{4}$$



(a) The brane's scale factor a (blue), the bulk's scale factor b (green), and $\left|G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}\right|$ (black) are plotted versus time for $0.02 < \tilde{\Lambda} < 0.03$. $a_0 = 1$, and $t_0 = 13.842$ [Gyr].



(b) The brane's scale factor (blue) fits the scale factor of the ΛCDM model (red) for the range $0.02 < \tilde{\Lambda} < 0.03$. The yellow curve shows the brane's scale factor for $\tilde{\Lambda} = -0.01$.

FIGURE 1

3. BRANE EMBEDDING AND THE FIELD EQUATIONS

We construct a 3-brane that may be thought of as a flat Robertson-Walker universe embedded in five dimensions. This is mapped by the metric

$$ds^2 = g_{MN}\,dx^M\,dx^N = -dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right) + b^2(t)\,dy^2, \tag{1}$$

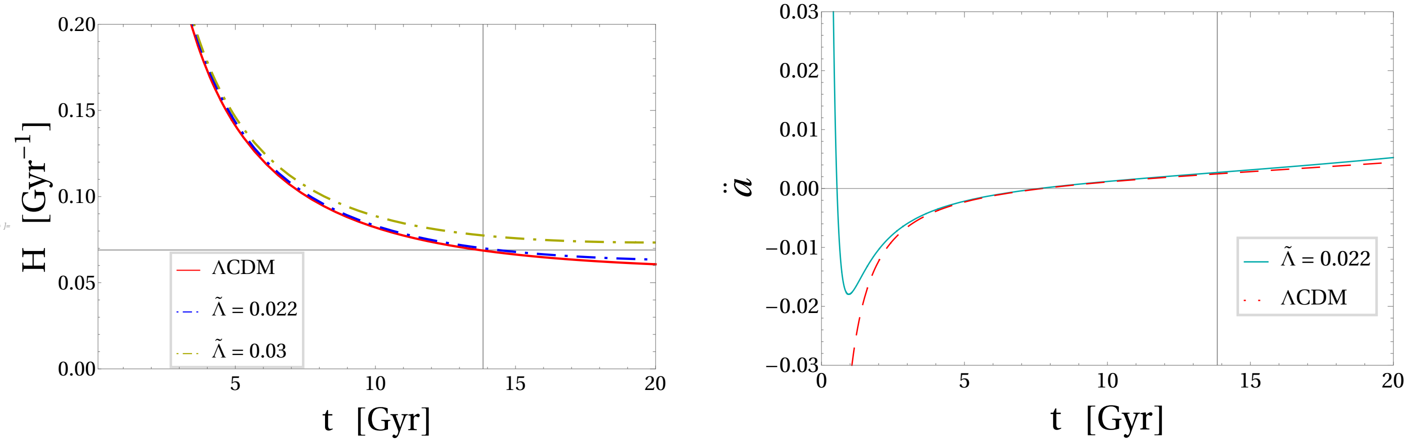
where $d\Omega^2 = d\theta^2 + \sin^2(\theta)\,d\varphi^2$, $a(t)$ is the usual Robertson-Walker scale factor, and $b(t)$ is a scale factor for the bulk dimension y. The brane is located at $y = 0$ and we ignore all possible y-dependence of the warp factors as well as of the hypermultiplet bulk fields. Consider the brane is filled by a perfect fluid, the following components of the brane's stress tensor are added to (4):

$$\begin{aligned}
T_{tt}^{\text{Brane}} &= \rho_m(t) + \rho_r(t) = \frac{\rho_{m0}}{a^3} + \frac{\rho_{r0}}{a^4} \\
T_{rr}^{\text{Brane}} &= a^2 p(t) = a^2\frac{\rho_{r0}}{3a^4} \\
T_{\theta\theta}^{\text{Brane}} &= a^2 r^2 p(t) = a^2 r^2\frac{\rho_{r0}}{3a^4}, \\
T_{\varphi\varphi}^{\text{Brane}} &= a^2 r^2\sin^2\theta\, p(t) = a^2 r^2\sin^2\theta\,\frac{\rho_{r0}}{3a^4}.
\end{aligned} \tag{2}$$

We consider the brane cosmological constant $\Lambda = 0$ and show how the late-time acceleration can be produced only from the moduli and the bulk cosmological constant $\tilde{\Lambda}$ effects. Then Einstein's equations $G_{MN} + \Lambda g_{MN} = \kappa T_{MN}$, where $\kappa = \frac{8\pi G}{c^4}$ give the following Friedmann-like equations:

$$\begin{aligned}
\left[\left(\frac{\dot{a}}{a}\right)^2 + \left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right)\right] &= G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}} + H_0^2\left(\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{r0}}{a^4}\right) \\
2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{\ddot{b}}{b} + 2\left(\frac{\dot{a}}{a}\right)\left(\frac{\dot{b}}{b}\right) &= -H_0^2\frac{\Omega_{r0}}{a^4} - G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}} \\
3\left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] &= \tilde{\Lambda} - G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}},
\end{aligned} \tag{3}$$

The first and the second equations represent the Friedmann-like equations for the brane-universe, while the third equation represents the bulk. Solving those equations numerically to get the brane's and the bulk's scale factors a and b, respectively, and the moduli's flow velocity $G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}$ [7]. We scan over a range of the bulk's cosmological constant $0.02 < \tilde{\Lambda} < 0.03$. Whilst the solutions are valid for a wide range of initial conditions fine-tuning. Here we take $a[0] = b[0] \sim 0.06$ and $a'[0] = b'[0] \sim 0.01$. In our model, we consider the value of the dark energy density parameter $\Omega_\Lambda = 0$, and the current matter density parameter $\Omega_{m0} \sim 0.99$, so in our model the main components of the brane-universe are only matter and radiation. In the ΛCDM model



(a) The Hubble parameter of ΛCDM (red), the brane's Hubble parameters for $\tilde{\Lambda} = 0.022$ [Gyr^{-2}] (blue dashed) and $\tilde{\Lambda} = 0.03$ [Gyr^{-2}] (yellow dashed) are plotted versus time. $H_0 = 0.0689751$ [Gyr^{-1}].

(b) The acceleration of the brane's scale factor (blue) and the acceleration of the scale factor of ΛCDM (red dashed) are plotted versus time. Around $t = 10$ [Gyr] the accelerated expansion has been initiated.

FIGURE 2

according to the recent Planck mission data based on CMB (The Cosmic Microwave Background) [8] $\Omega_{m0} = 0.3111$, the current radiation density parameter $\Omega_{r0} = 8.2 \times 10^{-5}$, and $\Omega_{\Lambda 0} = 0.6889$. The current value of the Hubble parameter is given by $H_0 = 0.0686751$ [Gyr^{-1}] $\sim 2.176 \times 10^{-18}$ [sec^{-1}]. The age of the universe is $t_0 = 13.842$ [Gyr]. That amount of Ω_{m0} in the brane-universe means that the current dark matter density parameter $\Omega_{Dm0} \sim 0.94$ in the brane because the current baryonic matter density parameter in our universe is $\Omega_{Bm0} = 0.0463 \pm 0.0024$. That huge amount of Ω_{Dm0} in the brane may affect other cosmological observable like the baryonic acoustic oscillation or the galactic rotations curves. However, next, we will show that the cosmic evolution of the brane-universe is similar to the expansion history of our universe according to BOSS collaboration data based on the baryon acoustic oscillation (BAO) and the CMB constraints [9]. In Fig. (1a) the brane's and the bulk's scale factors, and the absolute value of the moduli velocity norm are plotted versus time. Through these ranges $a[t_0] = 1$ as the scale factor of the ΛCDM model. The bulk's scale factor varies by different values and it is inversely proportional to $\tilde{\Lambda}$. Also the correlation between $G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}$ and $a(t)$ is clear, as the moduli are maximum $a(t)$ is minimum, and vise versa. Fig. (1b) shows that the scale factor of the brane (blue) fits and the ΛCDM's scale factor (red) over the scanned parameter region, where the current value of the ΛCDM's scale factor $a(t_0)$ is normalized to $a_0 = 1$. While for instance, an Anti-de Sitter bulk with a negative $\tilde{\Lambda}$ leads the brane-univesre for a deceleration. Fig. (2a) shows that brane's Hubble parameter (blue dashed) fits the Hubble parameter of the ΛCDM model (red) at $\tilde{\Lambda} = 0.022$. The horizontal line is at H_0. In Fig. (2b) the acceleration of the brane's scale factor at $\tilde{\Lambda} = 0.022$ (blue) and the acceleration of the scale factor of ΛCDM (red dashed) are plotted versus time. We can see the deceleration and the acceleration eras that happened after the big bang by $(3-5)$ Gyr and $(9-10)$ Gyr, respectively, according to the BOSS's data. Such that during the decelerated expansion the brane's acceleration was negative ($\ddot{a} < 0$), then around 10 Gyr ($\ddot{a} > 0$) till the present time (t_0). Through those epochs the brane and the ΛCDM's accelerations coincide, while in the early times, the brane's acceleration shows inflationary behavior ($\ddot{a} > 0$) which the ΛCDM model does not explain. In the next graphs, we will show clearly the very early ages of the expansion.

4. THE COSMIC HISTORY

In Fig. (3) the acceleration of the brane-universe is plotted versus time on a logarithmic scale. We can see that the early times are zoomed in, and the whole cosmic evolution of the universe is shown. The pink era is when the inflation started around $t \sim 10^{-53}$ [Gyr] and ended on $t \sim 10^{-43}$ [Gyr] where ($\ddot{a} > 0$). Then the dark green era when the CMB happened, the light green era when the decelerated expansion took place, and the light blue era corresponds to the late-time acceleration expansion. In Fig. (4) the Hubble parameter $H(a)/H_0$ for the brane (blue) and the ΛCDM model (red) are plotted as functions in the expansion scale factor (a/a_0) on a logarithmic scale ($\tilde{\Lambda} = 0.022$). The pink area corresponds to the inflation era, and the dark green area starts when the CMB happened after the big bang by $t_{CMB} = 380.000$ years and $a[t_{CMB}] \sim 2.3 \times 10^{-5}$ [Gyr]. The light green era corresponds to when the deceleration of the universe's expansion happened, and the light blue era corresponds to when the late-time acceleration happens after $(9-10)$ billion years of the big bang till time beings. That is according to the experimental data of our universe's expansion based on the baryon acoustic oscillation (BAO) reported by BOSS collaboration. We can see that $H(a)/H_0$ of the brane met that of the ΛCDM during the deceleration era. It is also shown that (a/a_0) through the inflation period where there are enough number of e-folds $N = ln(a_e/a_0)$ (e stands for the end of the inflation) to solve the Horizon problem. Let us draw the attention that the values of $\tilde{\Lambda}$ are given in [Gyr^{-2}]. For the sake of comparison, the value of Λ in ΛCDM is given by $\frac{8\pi G}{c^2}\,\rho_{\Lambda}^{obs} \sim 9.7 \times 10^{-36}$ sec^{-2} ~ 0.00969 Gyr^{-2}, while for instance $\tilde{\Lambda} = 0.022$ Gyr^{-2} $= 5.5 \times 10^{-35}$ sec^{-2}. Fig. (5) shows the relation of the moduli to the inflation slow-roll parameter $\epsilon = -\frac{\dot{H}}{H^2}$, where ϵ is plotted against the moduli's flow velocity $|\dot{z}|^2 \equiv \left(G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}\right)$

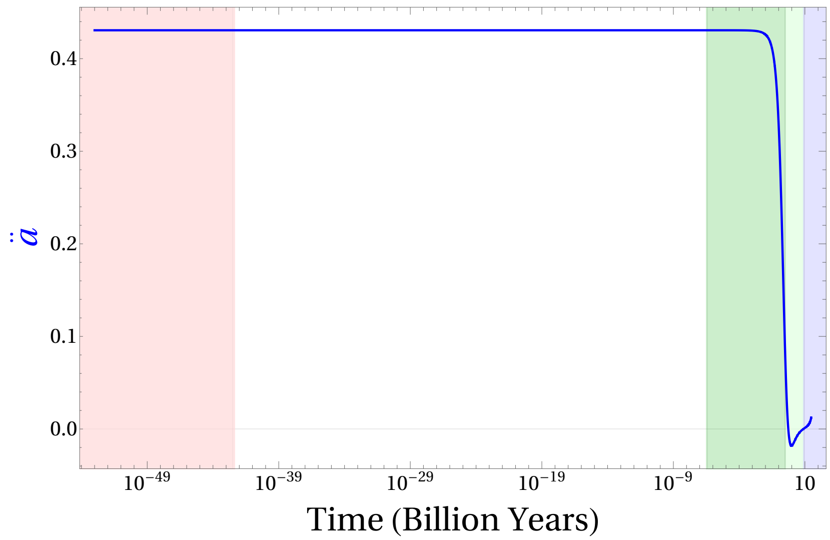


FIGURE 3: The acceleration of the brane's scale factor is plotted versus time on a logarithmic scale ($\tilde{\Lambda} = 0.022.$). The pink area corresponds to the inflation era, the dark green area when the CMB happens, the light green area when the deceleration expansion happened, and the light blue area corresponds to the late-time acceleration era.

at $\tilde{\Lambda} = 0.022$ (blue) and $\tilde{\Lambda} = 0.03$ (green). We can see that when the moduli have larger values $\epsilon < 1$ at inflation which means that H is varying slowly or there is an accelerated phase of expansion. Then when $|\dot{z}|^2$ decrease the inflation ends, where $\epsilon > 1$.

5. FINDING ANALYTIC SOLUTION

To find an analytic solution for the field equations (3), we consider the bulk has a constant size, i.e., the bulk's scale factor is constant ($\dot{b} = 0$). The field equations simplify to:

$$\begin{aligned} 3\left(\frac{\dot{a}}{a}\right)^2 &= G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}} + \rho, \\ 2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 &= -p - G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}, \\ \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right] &= \tilde{\Lambda} - G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}}. \end{aligned} \tag{1}$$

Eliminating $(G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}})$ gives:

$$\begin{aligned} p &= \frac{2\dot{a}^2}{a} + \frac{\ddot{a}}{a} + \tilde{\Lambda}, \\ \rho &= \frac{4\dot{a}^2}{a^2} + \frac{2\ddot{a}}{a} + p. \end{aligned} \tag{2}$$

Solving these couple of equations gives:

$$\begin{aligned} a(t) &= c_2 \cos\left[\sqrt{\frac{3}{2}}\sqrt{p-\rho}(t+2c_1)\right]^{1/3}, \\ &= c_2 \cos\left[\sqrt{3}\sqrt{\tilde{\Lambda}-p}(t-c_1)\right]^{1/3}. \end{aligned} \tag{3}$$

Where c_1 and c_2 are the integration constants. That sets a constrain on the density ρ and the pressure p in the brane, and $\tilde{\Lambda}$ in the bulk, $3p - 2\rho = \tilde{\Lambda}$. Another thing we want to declare about that class of models with a null cosmological constant ($\Lambda = 0$) [10], that they can prompt the string theory to be a theory of quantum gravity. If our universe as described in this paper has no cosmological constant, the Swampland conjectures do not apply to the string theory as a low-energy theory of our universe.

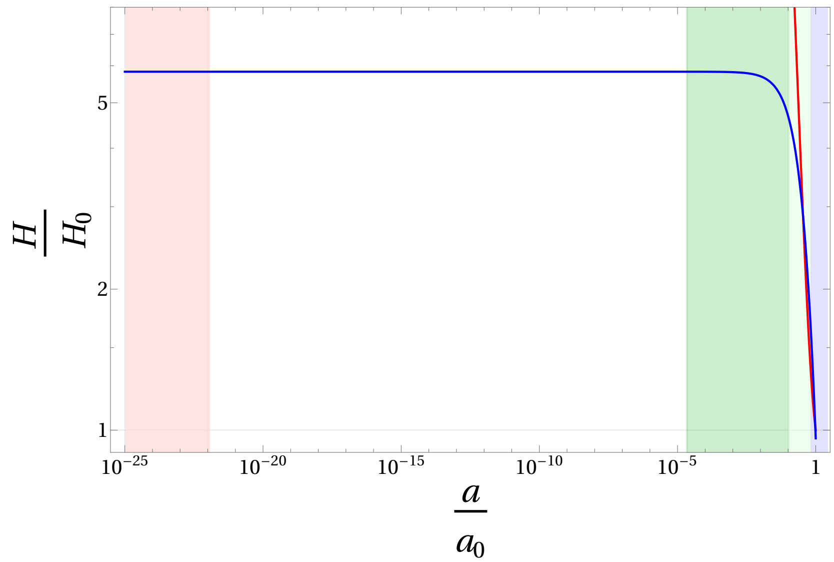


FIGURE 4: $H(a)/H_0$ for the brane-universe (blue) and the ΛCDM model (red) are plotted versus the expansion scale factor. The pink area is a high regime era where $H >> H_0$, which corresponds to the inflation era, and the dark green area is when the CMB happens. Then H/H_0 starts to get close to unity as the brane starts deceleration expansion through the light green area. The light blue area when the accelerated expansion started at $a \sim 0.7$ till the time beings.

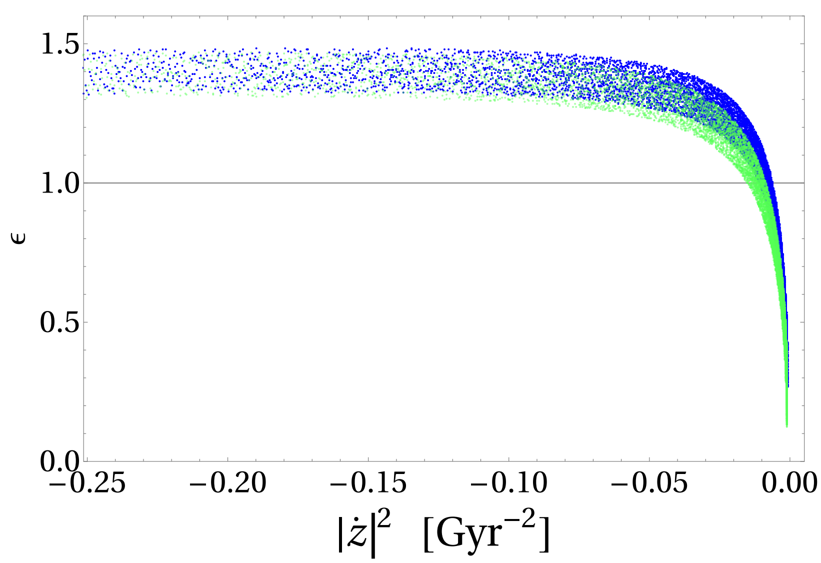


FIGURE 5: The slow-roll parameters are plotted against the moduli's flow velocity $(G_{i\bar{j}}\dot{z}^i\dot{z}^{\bar{j}})$ at $\tilde{\Lambda} = 0.022$ (blue) and $\tilde{\Lambda} = 0.03$ (green).

6. CONCLUSION

What is the origin of the late-time accelerated expansion of the universe? We find It difficult to accept that its origin is the cosmological constant in Einstein's general relativity because in GR the cosmological constant is related to the vacuum energy density which is tremendously large compared to the observed energy density (dark energy) that is responsible for the universe's accelerated expansion. In this paper, we introduced a new explanation for dark energy by modeling the universe as a symplectic 3-brane embedded in a five-dimensional bulk of $\mathcal{N} = 2$ supergravity. The brane-universe has an early time inflation, followed by deceleration, then followed by a late-time acceleration adequate to our universe's cosmic evolution. Although the 3-brane has a zero cosmological constant, solving the modified Friedmann's equations shows that the fields which drive the dynamics of the brane and the bulk are the complex structure moduli of the Calabi-Yau manifold. The bulk should be a di-Sitter space with a non-vanishing cosmological constant. The results correspond to the ΛCDM model and agree with the recent experimental data presented by BOSS collaboration about our universe's expansion history based on the baryon acoustic oscillation (BAO) combined with the CMB constraints.

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