Flavor and Lepton Universality Violation Phenomena in F-Theory Inspired GUTs

Athanasios Karozas,[1] George K. Leontaris,[1] Ilias Tavellaris,[1] and Nicholas D. Vlachos[2]
[1] *Physics Department, University of Ioannina 45110, Ioannina, Greece*
[2] *Department of Nuclear and Elementary Particle Physics, Aristotle University of Thessaloniki 54124, Thessaloniki, Greece*

Abstract

We discuss low energy implications of F-theory GUTs based on SU(5) extended by a U(1)′ symmetry which couples non-universally to the three chiral families. Several classes of anomaly free models are obtained, distinguished with respect to the U(1)′ charges of the representations, and possible extra zero modes coming in vector-like pairs. We considered the case where the spontaneous breaking of the U(1)′ symmetry occurs at a few TeV scale and we compute the observables of several flavor violation exotic processes in the effective theory. Particular cases interpreting the B-meson anomalies observed in LHCb experiments are also discussed.

Keywords: F-theory, flavor violation, lepton universality, LHCb anomalies
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1. INTRODUCTION

Over the last few years, hints of odd behaviour in the way B-mesons decay into Kaons have been continuously reported by the LHCb collaboration [1, 2, 3]. The observed deviations signal possible departures from Lepton Flavor Universality (LFU) according to which charged leptons have identical electroweak interaction and a universal flavor independent coupling with the Z-boson. Very recently using run-1 and run-2 data with integrated luminocity of $9\,fb^{-1}$ [4], LHCb collaboration has reported an updated measurement for R_K, i.e., the ratio of the branching fractions for the decays $B^+ \to K^+\mu^+\mu^-$ and $B^+ \to K^+e^+e^-$. The LHCb experiment announced a 3.1σ deviation of the Standard Model (SM) predictions for R_K

$$R^{[1.1,6]}_{K_{LHCb}} = 0.846^{+0.042+0.013}_{-0.039-0.012} \tag{1.1}$$

where the central value is equal to the previous result [3] but now appears with a reduced uncertainty.

Other current experimental results, also suggest that new physics phenomena are probably related with the muon decays [5, 6, 7, 8]. For example, the recent announcement [9] of a 4.2σ deviation in $(g-2)_\mu$ measurements strengthen this opinion. Since muons are involved in such processes quite often these days, rare lepton flavor violation reactions like $\mu^- \to e^-e^-e^+$, $\mu^- \to e\gamma$ and $\mu - e$ convention may be observed in related experiments in the near future [10]. Hence in any proposed model which attempts to explain the current observed discrepancies special attention to the muon sector and possible muon flavor violation processes must be devoted.

Among the many SM extensions that have been proposed so far, those with leptoquarks and/or non-universal Z' gauge bosons accompanied by vector-like exotic fermions seem to be the most promising ones [11, 12, 13]. Moreover, new gauge degrees of freedom, pairs of vector-like exotics and leptoquark fields appear naturally in the rich structures of String Theory inspired models. F-theory [14] in particular, which is the geometric version of IIB theory, provides a series of advantages: in particular, the gauge symmetry, the spectrum and other essential properties of the effective theory are determined in terms of the geometry of the compactification manifold. Some common preditions of these models are discrete symmetries, vector-like families, and novel abelian factors which could remain unbroken at low energies [15, 16].

In this talk we discuss *non-universal* $U(1)'s$ in F-theory inspired models [17, 18]. We perform a detailed classification of this kind of models that naturally appear in F-theory and we discuss their phenomenological implications at low energies with emphasis on flavor violation phenomena.

2. NON-UNIVERSAL $U(1)'$S FROM F-THEORY

F-theory is a non-perturbative formulation of type-IIB theory invariant under a $SL(2, Z)$ symmetry. The internal space of F-theory is an elliptically fibred complex manifold where the complex modulus of the elliptic fiber, the axion-dilaton field τ, is a combination of the two scalars C_0, ϕ of the IIB bosonic spectrum, $\tau = C_0 + ie^{-\phi}$. This way, F-theory is defined on the 12-dimensional background $R^{(3,1)} \times \mathcal{X}$ where $R^{(3,1)}$ is the four dimensional space-time and $\mathcal{X}$ an elliptically fibred Calabi-Yau complex fourfold with a section over a complex three-fold base B_3 (see Figure 1).

In F-theory, gauge degrees of freedom are linked to the singularities of the internal space. With respect to the algebraic description, the elliptic fibre is defined by the Weirstraß equation $y^2 = x^3 + fx + g$. Depending on the specific structure of its coefficients

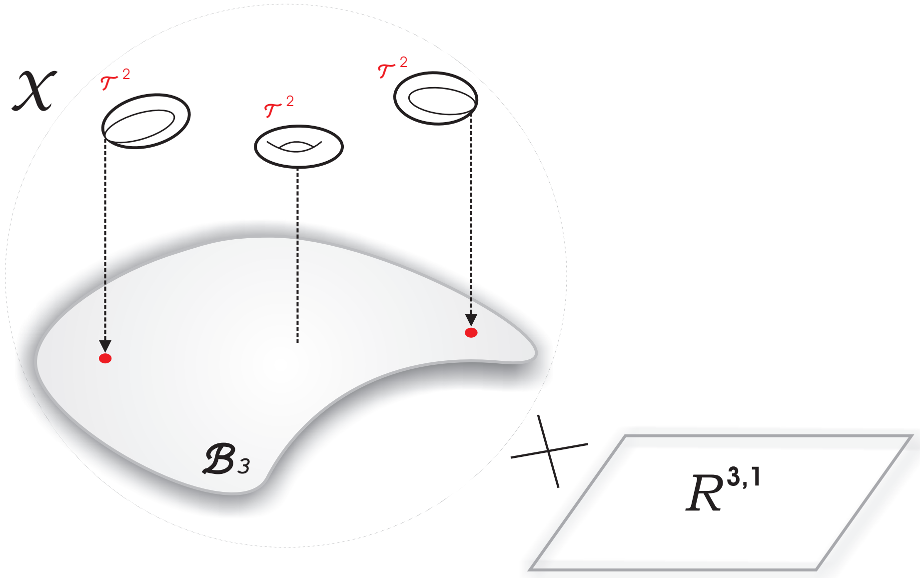

FIGURE 1: Graphic illustration of the total space in F-theory.

f, g, at certain points of the fibration the torus with modulus τ degenerates and the fibration becomes singular (the red points in Figure 1). It was shown long time ago [19] that these singularities can be classified in terms of ADE Lie groups with the highest one being the E_8 exceptional group which acts as parent symmetry for all the known GUT groups like E_6, $SO(10)$ and $SU(5)$. For reviews see [20, 21, 22] .

Here we will focus on a class of semi-local F-theory models with $SU(5) \times U(1)$ gauge symmetry obtained from the covering E_8 gauge group through the breaking sequence

$$E_8 \supset SU(5) \times SU(5)' \supset SU(5) \times U(1)^4 \supset SU(5) \times U(1)' \tag{2.1}$$

where $U(1)'$ represents some linear combination of the four abelian factors incorporated in $SU(5)'$.

The Cartan generators $Q_k = \mathrm{diag}\{t_1, t_2, t_3, t_4, t_5\}$, $k = 1, 2, 3, 4$ corresponding to the four $U(1)$ factors in (2.1), subjected to the $SU(5)$ tracelesness condition $\sum_{i=1}^{5} t_i = 0$, are taken to be

$$Q_a = \frac{1}{2}\mathrm{diag}(1, -1, 0, 0, 0), \qquad Q_b = \frac{1}{2\sqrt{3}}\mathrm{diag}(1, 1, -2, 0, 0), \tag{2.2}$$

$$Q_\psi = \frac{1}{2\sqrt{6}}\mathrm{diag}(1, 1, 1, -3, 0), \qquad Q_\chi = \frac{1}{2\sqrt{10}}\mathrm{diag}(1, 1, 1, 1, -4). \tag{2.3}$$

To ensure a tree-level top-quark mass a $\mathcal{Z}_2$ monodromy $t_1 \leftrightarrow t_2$ is imposed, "breaking" $U(1)_a$ while leaving invariant the remaining three abelian factors. In addition, appropriate fluxes can be turned on along the remaining $U(1)$'s in such a way that some linear combination $U(1)'$ of the abelian factors remains unbroken at low energies.

The $U(1)'$ assumed to be left unbroken at the effective model is a linear combination of the symmetries surviving the $\mathcal{Z}_2$ monodromy action, namely:

$$Q' = c_1 Q_b + c_2 Q_\psi + c_3 Q_\chi \,, \tag{2.4}$$

with the coefficients $c_{1,2,3}$ satisfying the normalization condition

$$\sum_{i=1}^{3} c_i^2 = 1 \,, \tag{2.5}$$

while further conditions to these coefficients will be imposed by applying anomaly cancellation conditions.

The $U(1)$ fluxes mentioned above, also determine the chiralities of the $SU(5)$ representations. Their effect on the representations of the various matter curves $\Sigma_{10_j}, \Sigma_{5_i}$ can be parametrized in terms of integers M_j, m_j as follows:

$$n_{10_i} - n_{\overline{10}_i} = m_i \qquad n_{5_j} - n_{\bar{5}_j} = M_j \tag{2.6}$$

while chirality is ensured by imposing the following condition

$$\sum_i m_i = -\sum_j M_j = 3 \,. \tag{2.7}$$

Furthermore, a second type of flux along the direction of hypercharge is turned on in order to break the $SU(5)_{GUT}$ group down to $SU(3)\times SU(2)\times U(1)_Y$ symmetry. Parametrizing this hypercharge flux with integers N_i, N_j the various multiplicities of the Standard Model representations are given by

$$10_{t_j} = \begin{cases} n_{(3,2)_{\frac{1}{6}}} - n_{(\bar{3},2)_{-\frac{1}{6}}} &= m_j \\ n_{(\bar{3},1)_{-\frac{2}{3}}} - n_{(3,1)_{\frac{2}{3}}} &= m_j - N_j \\ n_{(1,1)_{+1}} - n_{(1,1)_{-1}} &= m_j + N_j \end{cases} \;;\quad 5_{t_i} = \begin{cases} n_{(3,1)_{-\frac{1}{3}}} - n_{(\bar{3},1)_{+\frac{1}{3}}} &= M_i \\ n_{(1,2)_{+\frac{1}{2}}} - n_{(1,2)_{-\frac{1}{2}}} &= M_i + N_i \;. \end{cases} \tag{2.8}$$

Skipping further technical details, next we present only the properties of the general spectrum as shown in Table 1. For a complete analysis we refer to our work [18].

Matter Curve	Q'	N_Y	M	SM Content
$\Sigma_{10_{1,\pm t_1}}$	$\frac{10\sqrt{3}c_1+5\sqrt{6}c_2+3\sqrt{10}c_3}{60}$	$-N$	m_1	$m_1Q+(m_1+N)u^c+(m_1-N)e^c$
$\Sigma_{10_{2,\pm t_3}}$	$\frac{-20\sqrt{3}c_1+5\sqrt{6}c_2+3\sqrt{10}c_3}{60}$	N_7	m_2	$m_2Q+(m_2-N_7)u^c+(m_2+N_7)e^c$
$\Sigma_{10_{3,\pm t_4}}$	$\frac{\sqrt{10}c_3-5\sqrt{6}c_2}{20}$	N_8	m_3	$m_3Q+(m_3-N_8)u^c+(m_3+N_8)e^c$
$\Sigma_{10_{4,\pm t_5}}$	$-\sqrt{\frac{2}{5}}c_3$	N_9	m_4	$m_4Q+(m_4-N_9)u^c+(m_4+N_9)e^c$
$\Sigma_{5_{1,(\pm 2t_1)}}$	$-\frac{c_1}{\sqrt{3}}-\frac{c_2}{\sqrt{6}}-\frac{c_3}{\sqrt{10}}$	N	M_1	$M_1\overline{d^c}+(M_1+N)\overline{L}$
$\Sigma_{5_{2,\pm(t_1+t_3)}}$	$\frac{5\sqrt{3}c_1-5\sqrt{6}c_2-3\sqrt{10}c_3}{30}$	$-N$	M_2	$M_2\overline{d^c}+(M_2-N)\overline{L}$
$\Sigma_{5_{3,\pm(t_1+t_4)}}$	$-\frac{c_1}{2\sqrt{3}}+\frac{c_2}{\sqrt{6}}-\frac{c_3}{\sqrt{10}}$	$-N$	M_3	$M_3\overline{d^c}+(M_3-N)\overline{L}$
$\Sigma_{5_{4,\pm(t_1+t_5)}}$	$\frac{-10\sqrt{3}c_1-5\sqrt{6}c_2+9\sqrt{10}c_3}{60}$	$-N$	M_4	$M_4\overline{d^c}+(M_4-N)\overline{L}$
$\Sigma_{5_{5,\pm(t_3+t_4)}}$	$\frac{c_1}{\sqrt{3}}+\frac{c_2}{\sqrt{6}}-\frac{c_3}{\sqrt{10}}$	N_7+N_8	M_5	$M_5\overline{d^c}+(M_5+N_7+N_8)\overline{L}$
$\Sigma_{5_{6,\pm(t_3+t_5)}}$	$\frac{20\sqrt{3}c_1-5\sqrt{6}c_2+9\sqrt{10}c_3}{60}$	N_7+N_9	M_6	$M_6\overline{d^c}+(M_6+N_7+N_9)\overline{L}$
$\Sigma_{5_{7,\pm(t_4+t_5)}}$	$\frac{5\sqrt{6}c_2+3\sqrt{10}c_3}{20}$	N_8+N_9	M_7	$M_7\overline{d^c}+(M_7+N_8+N_9)\overline{L}$

TABLE 1: All the information for building $SU(5)\times U(1)$ models in F-theory semi-local approach. Here $N=N_7+N_8+N_9$.

The Table shows the $U(1)'$ charges, the flux data and the SM content of each matter curve. Using the data displayed in the Table it is straightforward to write down analytical expressions for the MSSM anomaly cancellation conditions and gauge anomaly conditions for the extra $U(1)$ factor, which have been analysed in detail in [18, 17]. It turns out that the MSSM anomaly cancellation conditions coincide with the chirality condition (2.7) imposed by the fluxes. On the other hand the anomaly cancellation conditions related to the $U(1)'$ factor are complicated expressions of the c_i-coefficients and the flux integers m_i, M_j and N_k. Hence, in order to solve for the c_i-coefficients and consequently to determine the $U(1)'$ charges we have to deal with the flux data first. At this point,

Singlet Field	Weights	Q'_{ij}	Multiplicity
θ_{13}	t_1-t_3	$\frac{\sqrt{3}c_1}{2}$	M_{13}
θ_{14}	t_1-t_4	$\frac{c_1+2\sqrt{2}c_2}{2\sqrt{3}}$	M_{14}
θ_{15}	t_1-t_5	$\frac{1}{12}\left(2\sqrt{3}c_1+\sqrt{6}c_2+3\sqrt{10}c_3\right)$	M_{15}
θ_{34}	t_3-t_4	$\frac{\sqrt{2}c_2-c_1}{\sqrt{3}}$	M_{34}
θ_{35}	t_3-t_5	$\frac{1}{12}\left(-4\sqrt{3}c_1+\sqrt{6}c_2+3\sqrt{10}c_3\right)$	M_{35}
θ_{45}	t_4-t_5	$\frac{1}{4}\left(\sqrt{10}c_3-\sqrt{6}c_2\right)$	M_{45}
θ_{31}	t_3-t_1	$-\frac{\sqrt{3}c_1}{2}$	M_{31}
θ_{41}	t_4-t_1	$-\frac{c_1+2\sqrt{2}c_2}{2\sqrt{3}}$	M_{41}
θ_{51}	t_5-t_1	$\frac{1}{12}\left(-2\sqrt{3}c_1-\sqrt{6}c_2-3\sqrt{10}c_3\right)$	M_{51}
θ_{43}	t_4-t_3	$\frac{c_1-\sqrt{2}c_2}{\sqrt{3}}$	M_{43}
θ_{53}	t_5-t_3	$\frac{1}{12}\left(4\sqrt{3}c_1-\sqrt{6}c_2-3\sqrt{10}c_3\right)$	M_{53}
θ_{54}	t_5-t_4	$\frac{1}{4}\left(\sqrt{6}c_2-\sqrt{10}c_3\right)$	M_{54}

TABLE 2: Singlet fields along with their corresponding $U(1)'$ charges Q'_{ij} and multiplicities M_{ij}. Notice that $Q'_{ij}=-Q'_{ji}$.

we also emphasize that singlet fields appearing in the present framework play an important role in the construction of realistic

F-theory models. Their properties are displayed in Table 2. The multiplicities and charges of these singlets are subject to anomaly cancellation and flatness conditions.

The precise determination of the spectrum in the present framework depends on the choice of the flux parameters. Up to now these parameters are subject only to the chirality constraint (2.7) however additional conditions can be imposed by demanding certain phenomenological properties of the effective model and a specific zero-mode spectrum. In what follows, we split our search in two types of spectra. Namely, minimal models which contain only the MSSM spectrum (without exotics), and models with MSSM spectrum plus pairs of vector-like fermions. In both cases the spectrum is accompanied by the singlet fields mentioned above.

For each scenario we put appropriate conditions on the flux parameters and then we scan for all the possible combinations of flux integers satisfying all the criteria. Next, each flux solution is applied to the $U(1)'$ anomaly cancellation conditions in order to solve for the c_i coefficients, always with respect to the normalization condition (2.5).

In the following sections we present the results of our scanning procedure and briefly discuss their phenomenological properties.

3. MODELS WITHOUT EXOTICS

We start our presentation with the minimal case where the models we are interested in have exactly three generations of quarks and leptons accompanied only by singlet fields and a non-universal Z' gauge boson associated with a $U(1)'$ symmetry assumed to be unbroken at a scale of a few tens of TeV.

In order to produce the desired spectrum we have to put appropriate conditions on the fluxes. While the three MSSM families of quarks and leptons are readily ensured by the chirality condition (2.7), additional phenomenological requirements can further restrict the flux parameter space. For example, a solution to the doublet-triplet splitting problem implies that

$$|N_7| + |N_8| + |N_9| \neq 0. \tag{3.1}$$

Another set of conditions descends from the requirement for a tree-level top Yukawa coupling. Notice from Table 1 that the only renormalisable up-type operator is $10_1 10_1 5_1$. This suggests the following conditions on some of the flux parameters

$$m_1 = 1,\ m_1 + N \geq 1,\ M_1 + N \geq 1, \tag{3.2}$$

and in order to further isolate only the up Higgs doublet H_u at 5_1 matter curve we assume that $M_1 = 0$ and $N = 1$. Finally, absence of exotics implies that

$$m_i \geq 0\ ,\ -M_j \geq 0\ . \tag{3.3}$$

Scanning the flux parameter space by allowing values in the range $[-3, 3]$ with respect to all the MSSM flux conditions discussed so far we have received a set of 54 flux integers solutions. These 54 solutions fall into four classes. A representative solution of the fluxes and the corresponding c_i coefficients for each class is given in Table 3. The spectrum of the corresponding model is given in Table 4. Each class consists of various flux and c_i solutions that result to the same $U(1)'$ charges. The various models inside a class differ on how the SM states are distributed on the various matter curves.

Model	m_1	m_2	m_3	m_4	M_1	M_2	M_3	M_4	M_5	M_6	M_7	N_7	N_8	N_9	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{c_3}$
A	1	2	0	0	0	0	0	0	-1	-1	-1	1	0	0	0	$-\frac{1}{2}\sqrt{\frac{3}{2}}$	$\frac{1}{2}\sqrt{\frac{5}{2}}$
B	1	0	1	1	0	-1	0	0	-1	0	-1	0	1	0	$-\frac{\sqrt{5}}{3}$	$\frac{1}{6}\sqrt{\frac{5}{2}}$	$-\frac{1}{2}\sqrt{\frac{3}{2}}$
C	1	0	0	2	0	0	-1	0	0	-1	-1	0	0	1	$-\frac{\sqrt{5}}{6}$	$\frac{7}{12}\sqrt{\frac{5}{2}}$	$-\frac{1}{4\sqrt{6}}$
D	1	1	0	1	0	0	0	0	-1	-1	-1	0	0	1	$\frac{1}{2}\sqrt{\frac{5}{6}}$	$\frac{5}{8}\sqrt{\frac{5}{3}}$	$-\frac{3}{8}$

TABLE 3: 'MSSM'-spectrum flux solutions along with the corresponding c_i 's.

We observe that the various SM states, in general appear with non-universal charges under the extra $U(1)$ factor. Since the 10_1 matter curve always accommodates the third generation up quark, we observe from Table 4 that at least one of the lightest left-handed quarks will have the same $U(1)'$ charge with top quark. Hence the corresponding flavor violating processes associated with these two generations are expected to be suppressed. On the other hand flavor violating processes related to the two lightest quark families (i.e Kaon oscillations) are expected to be dominant. This property holds for all the 54 models derived in this framework. As an example we analyse in some detail the phenomenological implications of model D.

3.1. Analysis of Model D

Details for the quark and lepton sector of this model are displayed in Table 4. In order to achieve realistic fermion hierarchies we assume the following distribution of the MSSM spectrum onto the various matter curves

$$10_1 \longrightarrow Q_3 + u^c_{2,3},\ \ 10_2 \longrightarrow Q_1 + u^c_1 + e^c_1,\ \ 10_4 \longrightarrow Q_2 + e^c_{2,3}\ ,$$

Curve	Model **A**		Model **B**		Model **C**		Model **D**	
	Q'	SM	$\sqrt{15}Q'$	SM	$\sqrt{15}Q'$	SM	$\sqrt{10}Q'$	SM
10_1	0	$Q+2u^c$	-1	$Q+2u^c$	1/4	$Q+2u^c$	3/4	$Q+2u^c$
10_2	0	$2Q+u^c+3e^c$	3/2	-	3/2	-	-1/2	$Q+u^c+e^c$
10_3	1/2	-	-1	$Q+2e^c$	-9/4	-	-7/4	-
10_4	-1/2	-	3/2	$Q+u^c+e^c$	1/4	$2Q+u^c+3e^c$	3/4	$Q+2e^c$
5_1	0	H_u	2	H_u	-1/2	H_u	-3/2	H_u
$\bar{5}_2$	0	d^c+2L	1/2	d^c+2L	7/4	L	1/4	L
$\bar{5}_3$	-1/2	L	-2	L	-2	d^c+2L	-1	L
$\bar{5}_4$	1/2	L	1/2	L	1/2	L	3/2	L
$\bar{5}_5$	-1/2	d^c	1/2	d^c	-3/4	-	-9/4	d^c+L
$\bar{5}_6$	1/2	d^c	3	d^c	7/4	d^c	1/4	d^c
$\bar{5}_7$	0	-	1/2	-	-2	d^c	-1	d^c

TABLE 4: Representative solutions of models without exotics.

$$5_1 \longrightarrow H_u,\ \bar{5}_2 \longrightarrow H_d,\ \bar{5}_3 \longrightarrow L_3,\ \bar{5}_4 \longrightarrow L_2,\ \bar{5}_5 \longrightarrow d^c_1 + L_1,\ \bar{5}_6 \longrightarrow d^c_2,\ \bar{5}_7 \longrightarrow d^c_3\ ,$$

where the indices in the SM fields denote generation.

The properties (charges and multiplicities) of the singlet fields sector of the model are summarized below:

$$\begin{aligned} &Multiplicities: \ M_{13}=M_{14}=M_{15}=M_{34}=M_{45}=M_{41}=M_{53},\ M_{51}=2,\ M_{31}=M_{43}=3,\ M_{35}=M_{54}=4\ ,\\ &Charges\times\sqrt{10}: \ Q'_{13}=Q'_{34}=Q'_{53}=5/4,\ Q'_{14}=Q'_{45}=-5/2,\ Q'_{15}=0 \end{aligned}$$

while the charges (not shown here) are subject to $Q'_{ij}=-Q'_{ji}$ (see Table 2).

Now we can write down the superpotential and in particular the various terms contributing to the fermion mass matrices. We start with the up-quark sector. The dominant contributions to the up-type quark masses descend from the following superpotential terms:

$$\begin{aligned} W \supset\ & y_t 10_1 10_1 5_1 + \frac{y_1}{\Lambda} 10_1 10_2 5_1 \theta_{13} + \frac{y_2}{\Lambda} 10_1 10_4 5_1 \theta_{15} + \frac{y_3}{\Lambda^2} 10_2 10_4 5_1 \theta_{13}\theta_{15} \\ & + \frac{y_4}{\Lambda^2} 10_2 10_2 5_1 \theta_{13}^2 + \frac{y_5}{\Lambda^2} 10_1 10_2 5_1 \theta_{15}\theta_{53} + \frac{y_6}{\Lambda^3} 10_2 10_2 5_1 \theta_{15}\theta_{53}\theta_{13}\ , \end{aligned} \tag{3.4}$$

where y_i's denote coupling constant coefficients and Λ is a characteristic high energy scale of the theory.

Next we analyse the couplings of the down-quark and charged lepton sectors. The dominant terms contribute to the down quark sector are:

$$\begin{aligned} W \supset\ & y_b 10_1 \bar{5}_7 \bar{5}_2 + \frac{\kappa_1}{\Lambda} 10_1 \bar{5}_5 \bar{5}_2 \theta_{53} + \frac{\kappa_2}{\Lambda} 10_1 \bar{5}_6 \bar{5}_2 \theta_{43} + \frac{\kappa_3}{\Lambda} 10_2 \bar{5}_7 \bar{5}_2 \theta_{13} + \frac{\kappa_4}{\Lambda^2} 10_2 \bar{5}_6 \bar{5}_2 \theta_{13}\theta_{43} \\ & + \frac{\kappa_5}{\Lambda^2} 10_2 \bar{5}_5 \bar{5}_2 \theta_{13}\theta_{53} + \frac{\kappa_6}{\Lambda^2} 10_2 \bar{5}_7 \bar{5}_2 \theta_{15}\theta_{53} + \frac{\kappa_7}{\Lambda^3} 10_2 \bar{5}_5 \bar{5}_2 \theta_{15}\theta_{53}^2 + \frac{\kappa_8}{\Lambda^3} 10_2 \bar{5}_6 \bar{5}_2 \theta_{14}\theta_{43}^2 + \frac{\kappa_9}{\Lambda} 10_4 \bar{5}_7 \bar{5}_2 \theta_{15} \\ & + \frac{\kappa_{10}}{\Lambda} 10_4 \bar{5}_5 \bar{5}_2 \theta_{13} + \frac{\kappa_{11}}{\Lambda^2} 10_4 \bar{5}_6 \bar{5}_2 \theta_{13}\theta_{45} + \frac{\kappa_{12}}{\Lambda^2} 10_4 \bar{5}_5 \bar{5}_2 \theta_{15}\theta_{53} + \frac{\kappa_{13}}{\Lambda^3} 10_4 \bar{5}_6 \bar{5}_2 \theta_{15}\theta_{45}\theta_{53}\ , \end{aligned} \tag{3.5}$$

where y_b and κ_i represent coupling constant coefficients. Note that the operators represented by the couplings κ_5, κ_7, κ_{10} and κ_{12} contribute also to the charged lepton sector. Additional contributions to the charged lepton sector of the model descend from the following superpotential terms

$$W \supset y_\tau 10_4 \bar{5}_3 \bar{5}_2 + \frac{\lambda_1}{\Lambda} 10_2 \bar{5}_4 \bar{5}_2 \theta_{43} + \frac{\lambda_2}{\Lambda} 10_2 \bar{5}_3 \bar{5}_2 \theta_{53} + \frac{\lambda_3}{\Lambda} 10_4 \bar{5}_4 \bar{5}_2 \theta_{45}\ , \tag{3.6}$$

where y_τ is a tree level Yukawa coupling and λ_i are general coupling constants.

When the Higgs fields and the various singlets θ_{ij} acquire vacuum expectation values (VEV), $\langle\theta_{ij}\rangle \neq 0$, they generate hierarchical non-zero entries in the mass matrices of quarks and charged leptons. These VEV's, however, are constrained by phenomenological requirements, such as the μ-term and the presence of possible R-parity violating (RPV) terms.

In the model under discussion the μ-term can be realized dynamically through the tree-level coupling $5_1\bar{5}_3\theta_{13}$. Clearly, to avoid decoupling of the Higgs doublets from the light spectrum, we must assume that $\langle\theta_{13}\rangle$ is very small and close to the TeV scale. Regarding RPV terms these can be controlled by imposing certain discrete symmetries of geometric origin [23, 24, 25] or by considering local Yukawa flux effects as described in [26].

In summary we obtain the following mass matrices for the up, down and charged lepton sector of the model:

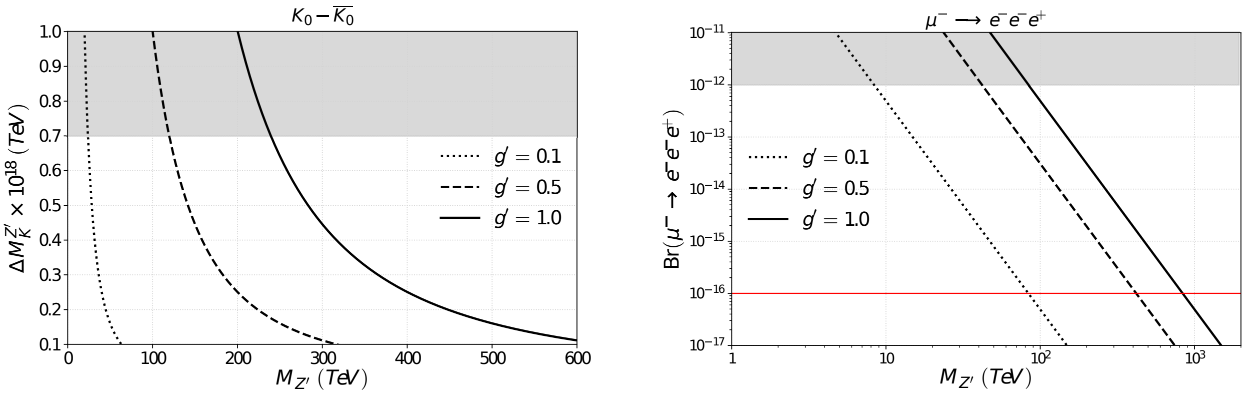

FIGURE 2: Bounds on the Z' gauge boson mass $M_{Z'}$ from the dominant flavor processes in model D. Left: Z' contributions to the mass split of the $K_0 - \overline{K}_0$ system. Right: Z' contributions to the branching ratio of the lepton flavor violation decay $\mu^- \to e^- e^- e^+$. The axes in this plot are in logarithmic scale. In both plots the grey shaded region excluded due to the current experimental bounds. The red horizontal line in the $\mu^- \to e^- e^- e^+$ plot represents the estimated reach of future lepton flavor violation experiments.

$$M_u = v_u \begin{pmatrix} y_4\vartheta_{13}^2 + y_6\vartheta_{15}\vartheta_{53}\vartheta_{13} & y_3\vartheta_{13}\vartheta_{15} & y_1\vartheta_{13} + y_5\vartheta_{15}\vartheta_{53} \\ y_1\vartheta_{13} + y_5\vartheta_{15}\vartheta_{53} & y_2\vartheta_{15} & \varepsilon y_t \\ y_1\vartheta_{13} + y_5\vartheta_{15}\vartheta_{53} & y_2\vartheta_{15} & y_t \end{pmatrix}, \tag{3.7}$$

$$M_d = v_d \begin{pmatrix} \kappa_5\vartheta_{53}\vartheta_{13} + \kappa_7\vartheta_{15}\vartheta_{53}^2 & \kappa_{10}\vartheta_{13} + \kappa_{12}\vartheta_{15}\vartheta_{53} & \kappa_1\vartheta_{53} \\ \kappa_4\vartheta_{13}\vartheta_{43} + \kappa_8\vartheta_{14}\vartheta_{43}^2 & \kappa_{11}\vartheta_{13}\vartheta_{45} + \kappa_{13}\vartheta_{15}\vartheta_{45}\vartheta_{53} & \kappa_2\vartheta_{43} \\ \kappa_3\vartheta_{13} + \kappa_6\vartheta_{15}\vartheta_{53} & \kappa_9\vartheta_{15} & y_b \end{pmatrix}, M_e = v_d \begin{pmatrix} \kappa_5\vartheta_{53}\vartheta_{13} + \kappa_7\vartheta_{15}\vartheta_{53}^2 & \lambda_1\vartheta_{43} & \lambda_2\vartheta_{53} \\ \kappa_{10}\vartheta_{13} + \kappa_{12}\vartheta_{15}\vartheta_{53} & \lambda_3\vartheta_{45} & \eta y_\tau \\ \kappa_{10}\vartheta_{13} + \kappa_{12}\vartheta_{15}\vartheta_{53} & \lambda_3\vartheta_{45} & y_\tau \end{pmatrix},$$

where $v_u = \langle H_u \rangle$, $v_d = \langle H_d \rangle$, $\vartheta_{ij} = \langle \theta_{ij} \rangle / \Lambda$ and ε , η are suppression factors introduced here to capture local Yukawa coupling effects of the theory [27, 28]. These matrices have the appropriate structure to produce the fermion mass hierarchies and due to the many parameters involved can easily reproduce the CKM matrix.

3.2. *Flavor violation effects and Z' bounds*

Since the fermions of the model appear with non-universal charges under the extra $U(1)'$ factor, the corresponding Z' boson surviving at low energies can have significant contributions to various flavor violation processes. The observables of these processes depend on the unitary matrices V_f diagonalizing the fermion mass matrices,on the charges of the various generations under the $U(1)'$ symmetry, on the mass of the extra gauge boson $M_{Z'}$ and on the gauge coupling g' of the extra $U(1)'$ symmetry [29, 30]. As mentioned before the strongest bounds are expected from $K - \overline{K}$ oscillation system.

First and second generation left-handed quarks appear with non-universal charges in the present model. Consequently, Z' contributes through tree-level interactions at the mass split of the $K - \overline{K}$ system. Using the formula for the mass split in neutral oscillation systems found in [29] our computations predict that the Z' contribution to the mass split of the Kaon system is:

$$\Delta M_K^{Z'} \approx 3.96 \times 10^{-14} \left(\frac{g' \; TeV}{M_{Z'}} \right)^2 .$$

The results are graphically presented in the left panel of Figure 2. The grey shaded region excluded due to the constraint [31]: $\Delta M_K^{NP} < 0.2 \Delta M_K^{exp}$ where $\Delta M_K^{exp} \simeq 3.482 \times 10^{-15}$ GeV [32]. To get an estimate, we observe that for $g' \approx 0.5$ (dashed curve) $M_{Z'} \gtrsim 120$ TeV which lies far above the most recent collider searches.

Important constraints descend also from lepton flavor violation processes. The dominant bound comes from the muon three body decay : $\mu^- \to e^- e^- e^+$. In particular our computation for the Z' contribution in branching ratio of the decay predicts that:

$$\mathrm{Br}(\mu^- \to e^- e^- e^+) \simeq 4.92 \times 10^{-5} \left(\frac{g' \; \mathrm{TeV}}{M_{Z'}} \right)^4 .$$

The result is plotted in terms of $M_{Z'}$ and for various values of g' on the right panel of Figure 2. The gray shaded region represents the current experimental bound [32], $\mathrm{Br}(\mu^- \to e^- e^- e^+) < 10^{-12}$. We observe that, for $g' \approx 0.5$ we receive $M_{Z'} \gtrsim 42$ TeV which is not compatible with the restrictions descending from the Kaon system.

However, the *Mu3e* experiment at PSI aims to improve the experimental sensitivity of the decay to $\sim 10^{-16}$ [10]. In the plot the red horizontal line represents the estimated reach of future $\mu \to 3e$ experiments. In this case , for $g' = 0.5$ we find that $M_{Z'} \gtrsim 420$ TeV in order the predicted branching ratio to satisfy the foreseen *Mu3e* experimental bounds. Hence, for the model under consideration, the currently dominant bounds from the $K - \overline{K}$ system will be exceeded in the near future (in absence of any result) by the limits of the upcoming $\mu^- \to e^- e^- e^+$ experiments.

Similar results are predicted for all the other models with an MSSM spectrum. It is clear from the analysis so far, that the strict constraints coming from the $K-\overline{K}$ system leave no room for a successful explanation of the R_K anomalies. A viable possibility to interpret the LHCb result within this framework, is to consider the class of models with vector-like quarks through their mixing with the conventional SM fermions [33]-[38].

4. MODELS WITH VECTOR LIKE EXOTICS

We turn our attention to models with the MSSM spectrum accompanied by vector-like (VL) states forming complete $(10+\overline{10})$ and $(5+\bar{5})$ pairs under the $SU(5)$ GUT symmetry. As in the previous analysis, we put appropriate conditions on the fluxes, solve the anomaly cancellation conditions, and derive the $U(1)'$ charges of all the models with additional VL fermion pairs.

Implementing all the appropriate flux restrictions, our scan returns 1728 flux solutions with one VL family in addition to the three chiral families of the SM.

This time in order to avoid strong restrictions from flavor violation processes we search for those models that appear with different $U(1)'$ charges for the VL states, while keeping universal the $U(1)'$ charges for the three SM fermion families. From the resulting 1728 models only 192 of them appear with the desired property. These 192 models fall into five classes with respect to their $SU(5)\times U(1)'$ properties. One model for each class is presented in Table 5.

Model **A′**		Model **B′**		Model **C′**		Model **D′**		Model **E′**	
$\sqrt{10}Q'$	SM	$\sqrt{85}Q'$	SM	Q'	SM	$\sqrt{10}Q'$	SM	$\sqrt{310}Q'$	SM
1/2	$Q+2u^c$	-2	$Q+2u^c$	1/4	$2Q+3u^c+e^c$	-3/4	$2Q+3u^c+e^c$	9/2	$Q+2u^c$
1/2	$2Q+u^c+3e^c$	-2	$2Q+u^c+3e^c$	-1/2	$Q+u^c+e^c$	-1/2	$\overline{Q}+\overline{u^c}+\overline{e^c}$	11/2	$\overline{Q}+\overline{u^c}+\overline{e^c}$
-2	$Q+u^c+e^c$	-1/2	$\overline{Q}+\overline{u^c}+\overline{e^c}$	1/4	$Q+2e^c$	7/4	$Q+u^c+e^c$	9/2	$2Q+u^c+3e^c$
-1/2	$\overline{Q}+\overline{u^c}+\overline{e^c}$	11/2	$Q+u^c+e^c$	1/4	$\overline{Q}+\overline{u^c}+\overline{e^c}$	-3/4	$Q+2e^c$	-8	$Q+u^c+e^c$
-1	H_u	4	H_u	-1/2	H_u	3/2	H_u	-9	H_u
1	d^c+2L	-4	L	-1/4	L	-1/4	d^c+2L	-1	L
-3/2	L	-3/2	L	1/2	L	1	L	-9	$\overline{d^c}$
1	L	7/2	L	0	$\overline{d^c}$	3/2	$\overline{d^c}$	-7/2	d^c+2L
-3/2	d^c	-3/2	d^c	-1/4	$3d^c+2L$	9/4	d^c+L	1	$\overline{L}$
1	$2d^c+L$	7/2	$3d^c+2L$	-3/4	d^c+L	-1/4	$2d^c+L$	-27/2	d^c+L
3/2	$\overline{d^c}+\overline{L}$	-6	$\overline{d^c}+\overline{L}$	0	$\overline{L}$	-1	$\overline{L}$	-7/2	$2d^c+L$

TABLE 5: Representative models containing vector-like exotics. The three MSSM families have universal $U(1)'$ charges and only the charges of the vector-like fields differ.

In these type of models, since the MSSM families appear with universal charges strong bounds from flavor violation processes can be evaded, while at the same time it is possible to explain the observed B meson anomalies through mixing effects of the VL exotics with the three MSSM chiral families. At the same time, RPV terms involving VL exotics which usually appear in these type of models, is an additional source of contributions in to $b\to s$ observables [39]-[43].

A complete classification of these type of models along with a detailed phenomenological analysis is underway and will be presented in a future publication.

5. CONCLUSIONS

In this work we have presented a class of models with $G_{51}=SU(5)\times U(1)'$ gauge symmetry derived in the framework of F-theory where the gauge symmetry of the effective theory is associated with the geometric singularities of the elliptically fibred compactification manifold. In this context, the gauge group of the present construction descends from the E_8 exceptional symmetry which is the highest smooth geometric singularity of the Kodaira classification [19]. A flux mechanism breaks the non-abelian (i.e., the $SU(5)$) part of the gauge group G_{51} at a high scale ($\sim 10^{16}$ GeV) down to the Standard Model gauge symmetry. At this stage, the $U(1)'$ factor is left intact by the fluxes and thusly remains unbroken down to low energies. A remarkable fact is that the corresponding gauge boson Z' displays non-universal gauge couplings to fermion families. A complete classification of the spectrum has been worked out and two main classes of models have been considered: those with the MSSM spectrum augmented only with neutral singlet fields and the Z' gauge boson, and those involving additional vector-like families having flavor mixing with the three ordinary chiral generations. The phenomenological properties of these effective theories have been investigated with particular emphasis to their predictions on flavor violating processed and on the B-meson anomalies reported by the LHCb collaboration. An analysis has been presented on whether the non-universal nature of the Z' couplings are capable of interpreting these effects. A generic observation is that the class of models with three chiral families only cannot explain the observed anomalous decays of B mesons. On the contrary, models with vector-like pairs of fermion generations exhibit flavor mixing with the ordinary families and as such, they can provide a reasonable explanation of the LHCb results, provided that the mass of the Z' boson is in the range of a few tens of TeV.

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References

[1] R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **111** (2013), 191801 doi:10.1103/PhysRevLett.111.191801 [arXiv:1308.1707 [hep-ex]].
[2] R. Aaij *et al.* [LHCb], JHEP **08** (2017), 055 doi:10.1007/JHEP08(2017)055 [arXiv:1705.05802 [hep-ex]].
[3] R. Aaij *et al.* [LHCb], Phys. Rev. Lett. **122** (2019) no.19, 191801 doi:10.1103/PhysRevLett.122.191801 [arXiv:1903.09252 [hep-ex]].
[4] R. Aaij *et al.* [LHCb], [arXiv:2103.11769 [hep-ex]].
[5] W. Altmannshofer and D. M. Straub, Eur. Phys. J. C **73** (2013), 2646 doi:10.1140/epjc/s10052-013-2646-9 [arXiv:1308.1501 [hep-ph]].
[6] R. Aaij *et al.* [LHCb], JHEP **02** (2016), 104 doi:10.1007/JHEP02(2016)104 [arXiv:1512.04442 [hep-ex]].
[7] V. Khachatryan *et al.* [CMS], Phys. Lett. B **753** (2016), 424-448 doi:10.1016/j.physletb.2015.12.020 [arXiv:1507.08126 [hep-ex]].
[8] R. Aaij *et al.* [LHCb], JHEP **05** (2020), 040 doi:10.1007/JHEP05(2020)040 [arXiv:1912.08139 [hep-ex]].
[9] B. Abi *et al.* [Muon g-2], Phys. Rev. Lett. **126** (2021) no.14, 141801 doi:10.1103/PhysRevLett.126.141801 [arXiv:2104.03281 [hep-ex]].
[10] F. Renga, Rev. Phys. **4** (2019), 100029 doi:10.1016/j.revip.2019.100029 [arXiv:1902.06291 [hep-ex]].
[11] S. Bifani, S. Descotes-Genon, A. Romero Vidal and M. H. Schune, J. Phys. G **46** (2019) no.2, 023001 doi:10.1088/1361-6471/aaf5de [arXiv:1809.06229 [hep-ex]].
[12] A. Cerri, V. V. Gligorov, S. Malvezzi, J. Martin Camalich, J. Zupan, S. Akar, J. Alimena, B. C. Allanach, W. Altmannshofer and L. Anderlini, *et al.* CERN Yellow Rep. Monogr. **7** (2019), 867-1158 doi:10.23731/CYRM-2019-007.867 [arXiv:1812.07638 [hep-ph]].
[13] K. Kowalska, D. Kumar and E. M. Sessolo, Eur. Phys. J. C **79** (2019) no.10, 840 doi:10.1140/epjc/s10052-019-7330-2 [arXiv:1903.10932 [hep-ph]].
[14] C. Vafa, "Evidence for F theory," Nucl. Phys. B **469** (1996) 403 doi:10.1016/0550-3213(96)00172-1 [hep-th/9602022].
[15] C. Beasley, J. J. Heckman and C. Vafa, "GUTs and Exceptional Branes in F-theory - I," JHEP **0901** (2009) 058 doi:10.1088/1126-6708/2009/01/058 [arXiv:0802.3391 [hep-th]];
[16] C. Beasley, J. J. Heckman and C. Vafa, "GUTs and Exceptional Branes in F-theory - II: Experimental Predictions," JHEP **0901** (2009) 059 doi:10.1088/1126-6708/2009/01/059 [arXiv:0806.0102 [hep-th]].
[17] M. Crispim Romao, S. F. King and G. K. Leontaris, Phys. Lett. B **782** (2018), 353-361 doi:10.1016/j.physletb.2018.05.057 [arXiv:1710.02349 [hep-ph]].
[18] A. Karozas, G. K. Leontaris, I. Tavellaris and N. D. Vlachos, Eur. Phys. J. C **81** (2021) no.1, 35 doi:10.1140/epjc/s10052-020-08794-y [arXiv:2007.05936 [hep-ph]].
[19] K. Kodaira, "On Compact Analytic Surfaces II," Annals Math. 77 (1963) 563
[20] T. Weigand, PoS TASI **2017** (2018) 016 [arXiv:1806.01854 [hep-th]].
[21] J. J. Heckman, Ann. Rev. Nucl. Part. Sci. **60** (2010) 237 [arXiv:1001.0577].
[22] G. K. Leontaris, PoS CORFU **2011** (2011) 095 [arXiv:1203.6277].
[23] I. Antoniadis and G. K. Leontaris, JHEP **08** (2012), 001 doi:10.1007/JHEP08(2012)001 [arXiv:1205.6930 [hep-th]].
[24] M. Crispim Romão, A. Karozas, S. F. King, G. K. Leontaris and A. K. Meadowcroft, Phys. Rev. D **93** (2016) no.12, 126007 doi:10.1103/PhysRevD.93.126007 [arXiv:1512.09148 [hep-ph]].
[25] Y. Kimura, JHEP **03** (2020), 153 doi:10.1007/JHEP03(2020)153 [arXiv:1908.06621 [hep-th]].
[26] M. Crispim Romão, A. Karozas, S. F. King, G. K. Leontaris and A. K. Meadowcroft, JHEP **11** (2016), 081 doi:10.1007/JHEP11(2016)081 [arXiv:1608.04746 [hep-ph]].
[27] S. Cecotti, M. C. N. Cheng, J. J. Heckman and C. Vafa, [arXiv:0910.0477 [hep-th]].
[28] G. K. Leontaris and G. G. Ross, JHEP **02** (2011), 108 doi:10.1007/JHEP02(2011)108 [arXiv:1009.6000 [hep-th]].
[29] P. Langacker and M. Plumacher, Phys. Rev. D **62** (2000), 013006 doi:10.1103/PhysRevD.62.013006 [arXiv:hep-ph/0001204 [hep-ph]].
[30] P. Langacker, Rev. Mod. Phys. **81** (2009), 1199-1228 doi:10.1103/RevModPhys.81.1199 [arXiv:0801.1345 [hep-ph]].
[31] C. H. Chen and T. Nomura, JHEP **03** (2019), 009 doi:10.1007/JHEP03(2019)009 [arXiv:1808.04097 [hep-ph]].
[32] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98** (2018) no.3, 030001 doi:10.1103/PhysRevD.98.030001
[33] S. M. Boucenna, A. Celis, J. Fuentes-Martin, A. Vicente and J. Virto, Phys. Lett. B **760** (2016), 214-219 doi:10.1016/j.physletb.2016.06.067 [arXiv:1604.03088 [hep-ph]].
[34] B. Allanach, F. S. Queiroz, A. Strumia and S. Sun, Phys. Rev. D **93** (2016) no.5, 055045 [erratum: Phys. Rev. D **95** (2017) no.11, 119902] doi:10.1103/PhysRevD.93.055045 [arXiv:1511.07447 [hep-ph]].
[35] S. F. King, JHEP **08** (2017), 019 doi:10.1007/JHEP08(2017)019 [arXiv:1706.06100 [hep-ph]].
[36] S. F. King, JHEP **09** (2018), 069 doi:10.1007/JHEP09(2018)069 [arXiv:1806.06780 [hep-ph]].
[37] J. Kawamura, S. Raby and A. Trautner, Phys. Rev. D **101** (2020) no.3, 035026 doi:10.1103/PhysRevD.101.035026 [arXiv:1911.11075 [hep-ph]].
[38] F. F. Freitas, J. Gonçalves, A. P. Morais and R. Pasechnik, JHEP **01** (2021), 076 doi:10.1007/JHEP01(2021)076 [arXiv:2010.01307 [hep-ph]].
[39] W. Huang and Y. L. Tang, Phys. Rev. D **92** (2015) no.9, 094015 doi:10.1103/PhysRevD.92.094015 [arXiv:1509.08599 [hep-ph]].
[40] K. Earl and T. Grégoire, JHEP **08** (2018), 201 doi:10.1007/JHEP08(2018)201 [arXiv:1806.01343 [hep-ph]].
[41] Q. Y. Hu, Y. D. Yang and M. D. Zheng, Eur. Phys. J. C **80** (2020) no.5, 365 doi:10.1140/epjc/s10052-020-7940-8 [arXiv:2002.09875 [hep-ph]].
[42] W. Altmannshofer, P. S. B. Dev, A. Soni and Y. Sui, Phys. Rev. D **102** (2020) no.1, 015031 doi:10.1103/PhysRevD.102.015031 [arXiv:2002.12910 [hep-ph]].
[43] S. Trifinopoulos, Eur. Phys. J. C **78** (2018) no.10, 803 doi:10.1140/epjc/s10052-018-6280-4 [arXiv:1807.01638 [hep-ph]].