A Self-Complementary Neutrino Mixing Model

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Abstract

We construct the self-complementary (SC) neutrino mixing pattern from the SC relation plus $\delta_{\rm CP} = -\frac{\pi}{2}$ and show that the indicated effective neutrino mass matrix has to be constructed perturbatively. We build an S_4 model for neutrino masses and mixings based on the SC neutrino mixing pattern. After performing a numerical study on the model's parameter space, we find that the model is phenomenologically viable in the case of normal ordering, and it gives predictions for the not-yet observed quantities like the lightest neutrino mass $m_1 \in [0.003, 0.010]$ eV and the Dirac CP violating phase $\delta_{\rm CP} \in [256.72^\circ, 283.33^\circ]$, which can be tested in the future experiments.

Keywords: Neutrino mixing, Flavor symmetry, Self-complementarity
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1. INTRODUCTION

Among the various attempts to understand the known fact that neutrinos are massive and have mixing, discrete flavour symmetries advantage in having a vivid physical picture and offering predictions in mixing and/or masses (see e.g. [1, 2] for reviews). In the meantime, phenomenological observations can give us hints of underlying structure, as they did for inspiring the discrete flavour symmetry in the beginning.

In this work, we build a neutrino mixing model based on a not-so-much investigated phenomenological relation: the self-complementarity relation (SC) of lepton mixing [3, 4], which is

$$\theta_{12} + \theta_{13} = \theta_{23} = 45^\circ, \tag{1}$$

where θ_{ij} are lepton mixing angles in the standard parametrization. It is observed in 2012 in light of the relatively large value of θ_{13} measured by reactor neutrino experiments and it correlates the three lepton mixing angles in a simple way.

This proceeding is based on work [5]. Here we briefly describe the model building procedure and then present the main results.

2. PREPARATION

To construct a mixing model from bottom-up, we need a mixing pattern featuring the self-complementary mixing first. Besides the relation $\theta_{12} + \theta_{13} = \theta_{23} = 45^\circ$, we also adopt a maximal CP-violating phase: $\delta_{\rm CP} = -\frac{\pi}{2}$. We use it for two reasons: firstly, it is the value of $\delta_{\rm CP}$ which is indicated by T2K [6] and NOνA [7], and is within the 1σ range of the global fit [8]; secondly, it is special in the sense that $\delta_{\rm CP}$ contributes its maximal to the Jarlskog invariant. Applying the SC relation together with $\delta_{\rm CP} = -\frac{\pi}{2}$ to the standard parameterization, we get the self-complementary mixing directly as

$$U_{\rm SC} = \begin{pmatrix} \cos\left(\frac{\pi}{4} - \theta_{13}\right)\cos\theta_{13} & \sin\left(\frac{\pi}{4} - \theta_{13}\right)\cos\theta_{13} & i\sin\theta_{13} \\ \frac{i\cos\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13} - \sin\left(\frac{\pi}{4}-\theta_{13}\right)}{\sqrt{2}} & \frac{\cos\left(\frac{\pi}{4}-\theta_{13}\right) + i\sin\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13}}{\sqrt{2}} & \frac{\cos\theta_{13}}{\sqrt{2}} \\ \frac{\sin\left(\frac{\pi}{4}-\theta_{13}\right) + i\cos\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13}}{\sqrt{2}} & \frac{i\sin\left(\frac{\pi}{4}-\theta_{13}\right)\sin\theta_{13} - \cos\left(\frac{\pi}{4}-\theta_{13}\right)}{\sqrt{2}} & \frac{\cos\theta_{13}}{\sqrt{2}} \end{pmatrix}, \tag{2}$$

and the whole lepton mixing matrix is $U_{\rm PMNS} = U_{\rm SC}.P,\ \ P = {\rm Diag}\{e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, 1\}$ when neutrinos are Majorana particles.

We can see that the SC mixing satisfies $|U_{\rm SC}|_{\cdot i} = |U_{\rm SC}|_{\tau i}$, which is the μ-τ exchange symmetry prediction for the mixing [9]. A mass matrix given by Eq. (2), which is at the meantime simple enough for model building, only reflects the two input: $\theta_{23} = 45^\circ,\ \delta_{\rm CP} = -\frac{\pi}{2}$. That is to say, the other ingredient of SC mixing, i.e., $\theta_{12} + \theta_{13} = 45^\circ$ is obscured from the direct construction of a mass matrix. This ingredient gives substructure of a mass matrix that is given by $\theta_{23} = 45^\circ,\ \delta_{\rm CP} = -\frac{\pi}{2}$. To see this substructure, we have to construct the mass matrix perturbatively.

By identifying $\sin\theta_{13} = \lambda$, $\cos\theta_{13} \cong 1 - \frac{1}{2}\lambda^2$, we get the expansion of $U_{\rm SC}$ in powers of λ,

$$
\begin{aligned}
U_{\rm SC} &\equiv U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + ... \\
&= \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & i \\ \frac{1}{2}+\frac{i}{2} & \frac{1}{2}+\frac{i}{2} & 0 \\ -\frac{1}{2}+\frac{i}{2} & -\frac{1}{2}+\frac{i}{2} & 0 \end{pmatrix} + \lambda^2 \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{4}+\frac{i}{2} & -\frac{1}{4}-\frac{i}{2} & -\frac{1}{2\sqrt{2}} \\ -\frac{1}{4}+\frac{i}{2} & \frac{1}{4}-\frac{i}{2} & -\frac{1}{2\sqrt{2}} \end{pmatrix} + ...
\end{aligned}
\tag{3}
$$

We use the expansion of $U_{\rm SC}$ to get the Majorana mass matrix expanded in λ,

$$
\hat{m}_\nu = U^*_{\rm SC}\hat{m}^d U^\dagger_{\rm SC} = (U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + ...)^* \hat{m}^d (U_{\lambda^0} + \lambda U_{\lambda^1} + \lambda^2 U_{\lambda^2} + ...)^\dagger \equiv \hat{m}_0 + \lambda\hat{m}_1 + \lambda^2\hat{m}_2 + ..., \tag{4}
$$

where $\hat{m}^d = diag\{m_1, m_2, m_3\}$. We use a hat notation above m to distinguish matrix $\hat{m}_i$ from eigenvalues m_i. Using the U_{λ^i} in Eq. (3), the mass matrix can be constructed accordingly. Since $\lambda = \sin\theta_{13} \simeq 0.15$, we will stop at $\mathcal{O}(\lambda^2)$, so the deviation from the exact SC mixing at percent level is expected. In next section, we will build a model to reproduce the neutrino mass matrix structure to this order.

3. THE MODEL

We list the field representations in S_4 and charges under additional symmetries in our model in the following table. Higgs is the singlet in S_4 and is not charged under any of the additional symmetries, so it is omitted in the table. The U(1) charges are arranged in a way that no new terms at the discussed order will show up, explicitly, $x \neq m \neq n \neq z$.

	L	e_R	μ_R	τ_R	N	ϕ_e	ϕ_μ	ϕ_τ	θ	ξ_1	ϕ_1	ψ_1	ϕ_{21}	ϕ_{22}	ϕ_{23}	ϕ_{31}	ϕ_{32}	ψ_3	ξ_3
S_4	3	1	1	1	3	3	3	3	1	1	3	2	3	3	3	3	3	2	1
U(1)	-x	z	m	n	x	$\frac{1}{2}(x-z)$	x-m	x-n	0	-2x	-2x	-2x	-x	-x	-x	$-\frac{2}{3}x$	$-\frac{2}{3}x$	y	-2x-2y
$\mathrm{U(1)_{FN}}$	0	2	1	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	0	0
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
$\mathbb{Z}_2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
$\mathbb{Z}_3$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	0	0

The effective Lagrangian reads

$$
\begin{aligned}
-\mathcal{L} &= y_e \bar{L}\tilde{H}e_{\rm R}\left(\frac{\phi_e}{\Lambda}\right)^2\left(\frac{\theta}{\Lambda}\right)^2 + y_\mu \bar{L}\tilde{H}\mu_{\rm R}\left(\frac{\phi_\mu}{\Lambda}\right)\left(\frac{\theta}{\Lambda}\right) + y_\tau \bar{L}\tilde{H}\tau_{\rm R}\left(\frac{\phi_\tau}{\Lambda}\right) + y_\nu \bar{L}HN \\
&+ \frac{y_{11}}{\Lambda}(NN)_1\xi_1 + \frac{y_{12}}{\Lambda}(NN)_2\psi_1 + \frac{y_{13}}{\Lambda}(NN)_3\phi_1 \\
&+ \frac{y_{21}}{\Lambda^2}(N\phi_{21})_3(N\phi_{21})_3 + \frac{y_{22}}{\Lambda^2}(NN)_3(\phi_{22}\phi_{22})_3 + \frac{y_{23}}{\Lambda^2}(NN)_3(\phi_{23}\phi_{23})_3 \\
&+ \frac{y_{31}}{\Lambda^3}(N\psi_3)_3(N\psi_3)_3\xi_3 + \frac{y_{32}}{\Lambda^3}\left((NN)_3\phi_{31}\right)_1(\phi_{31}\phi_{31})_1 + \frac{y_{33}}{\Lambda^3}\left((NN)_3\phi_{32}\right)_1(\phi_{32}\phi_{32})_1,
\end{aligned}
\tag{5}
$$

where the couplings y_{ij} in the Majorana neutrino mass term are of mass dimension 1 and are at the scale of heavy neutrino mass (we use a notation "y" instead of "m" to avoid confusion with various m in this model); the Λ denotes the cutoff scale of the theory.

The model constructed in this way that the structure of the neutrino mass matrix comes solely from S_4 breaking flavon vevs, which is derived from minimization of the scalar potential as is shown in Appendix of [5]. The $\mathbb{Z}_n$ symmetries are needed to distinguish the copies of flavons in the same S_4 representation, e.g., forbiding terms like $NN\phi_{21}\phi_{22}$. The U(1) symmetry forbid terms like $\bar{L}\tilde{H}e_R\phi_\mu\theta^2$. The $\mathrm{U(1)_{FN}}$ symmetry is responsible for the hierarchical masses of charged leptons. The potential Goldstone boson coming from the spontaneous breaking of U(1) symmetry may be gauged away by adding more particles, which is beyond the scope of the current work. It is also possible to use more cyclic symmetries instead of the U(1) symmetry to complete the same construction.

After the flavons in the charged lepton sector acquire vevs, we get a diagonal charged lepton mass matrix. Since the right handed charged leptons are all singlets of S_4, the resulting charged lepton mass matrix will always be diagonal (even when higher order operators enter). It gives no corrections to neutrino mixing and hence we omit it hereafter.

When the flavons in the neutrino sector acquire vevs, we get the following mass matrix for the heavy neutrinos

$$
\hat{m}_{\rm LO} = \frac{y_{11}}{\Lambda}v_{\xi_1}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + \frac{y_{12}}{\Lambda}v_{\psi_1}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{6} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{6} \end{pmatrix}\frac{y_{13}}{\Lambda}v_{\phi_1}\begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{6}
$$

$$
\hat{m}_{\rm NLO} = \frac{y_{21}}{\Lambda^2}v^2_{\phi_{21}}\begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{y_{22}}{\Lambda^2}v^2_{\phi_{22}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} + \frac{y_{23}}{\Lambda^2}v^2_{\phi_{23}}\begin{pmatrix} 0 & -2 & -2 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}, \tag{7}
$$

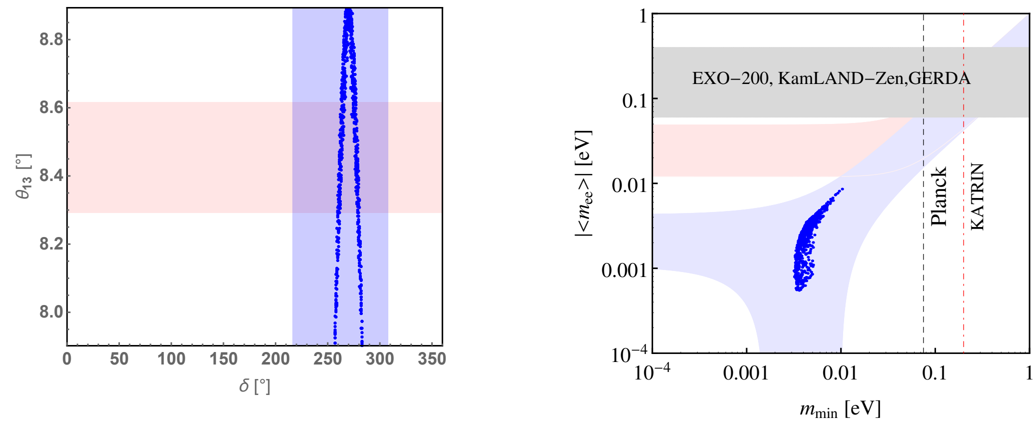

FIGURE 1: Numerical parameter space scan result. Blue points given by the model are in agreement with 3σ ranges of the low energy neutrino masses and mixing parameters. Left: The colored bands mark the 1σ ranges for θ_{13} and δ, the plot is framed in their 3σ ranges taken from Ref. [8]. Right: Prediction for the effective Majorana neutrino mass $|\langle m_{ee}\rangle|$ in neutrinoless double beta decay experiments as a function of the lightest neutrino mass m_{min}. The light blue (pink) region are obtained from the 3σ ranges of the low energy neutrino masses and mixings in case of normal (inverted) ordering. The vertical black dashed line is the Planck limit [11], and the vertical red dot-dashed line represents the limit on m_{min} (~ 0.2 eV) obtained from KATRIN sensitivity [12]. The light grey region is the upper limit on $|\langle m_{ee}\rangle|$ given by the EXO-200 [13], KamLAND-Zen [14], and GERDA experiments [15].

$$\hat{m}_{\text{NNLO}} = \frac{y_{31}}{\Lambda^3} v_{\psi_3}^2 v_{\xi_3} \begin{pmatrix} 3 & 0 & 0 \\ 0 & \frac{3}{2} & \frac{3}{2} \\ 0 & \frac{3}{2} & \frac{3}{2} \end{pmatrix} + \frac{y_{32}}{\Lambda^3} v_{\phi_{31}}^3 \begin{pmatrix} 0 & -2 & 2 \\ -2 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix} + \frac{y_{33}}{\Lambda^3} v_{\phi_{32}}^3 \begin{pmatrix} 0 & -2 & -2 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{pmatrix}. \tag{8}$$

Due to the strict constraints given by all the symmetries of the model, we find no higher order terms contributing to the Majorana neutrino mass matrix to mass dimension 10.

The effective light neutrino mass matrix is given by seesaw as

$$\hat{m}_{\nu_{\text{model}}} = \hat{m}_{\text{D}}(\hat{m}_{\text{LO}} + \hat{m}_{\text{NLO}} + \hat{m}_{\text{NNLO}} + ...)^{-1}\hat{m}_{\text{D}}^T \equiv \hat{m}_{\text{D}}\hat{m}_{\text{R}}^{-1}\hat{m}_{\text{D}}^T, \tag{9}$$

where we define $\hat{m}_{\text{R}} = \hat{m}_{\text{LO}} + \hat{m}_{\text{NLO}} + \hat{m}_{\text{NNLO}} + ...$ as the heavy neutrino mass matrix. It is expected from the above construction that $\hat{m}_{\text{R}}$ resembles the same structure inspired from the SC mixing to the order λ^2.

4. RESULTS AND CONCLUSION

The parameters in $\hat{m}_{\text{R}}$ can be further simplified. In the end we arrive at five independent real parameters in $\hat{m}_{\text{R}}$: $\tilde{v}_{\xi_1}$, $\tilde{v}_{\psi_1}$ and their phase γ; $\tilde{v}_{\phi_1}$ and its phase ρ. We perform a numerical scan of the parameter space. The seesaw scale is fixed to 10^{13} GeV and y_ν is taken to be 0.1. We use the REAP package [10] to perform the evolution of the mixing parameters from the seesaw scale to the low energy scale to make comparison with the oscillation observables.

In Figure 1 we show the result of the numerical parameter space scan for neutrino masses in the normal ordering. We can see that there are points given by the model in agreement with 3σ ranges of the low energy neutrino masses and mixing parameters. Given the fact that $\hat{m}_{\nu_{\text{model}}}$ cannot be of exactly the SC form, we see that the effects of $\hat{m}_{\text{D}}$ together with the RG running, and more importantly, the elaborately constructed $\hat{m}_{\text{R}}$, render a phenomenologically viable $\hat{m}_{\nu_{\text{model}}}$ in the low energy. It requires more work to disentangle these effects. We can get a sense of the RG effect by inputting the best fit values of the model parameters to the observables, and then performing a RG running down to the low energy. We find that, e.g. θ_{12} diminishes at a level of $\mathcal{O}(10^{-6})$ radian, θ_{13} and θ_{23} diminishes at a level of $\mathcal{O}(10^{-5})$ radian. This means a mild destructive effect. As we know what $\hat{m}_{\text{R}}$ would give, we conclude that the $\hat{m}_{\text{D}}$ effect is also mild and compensates the RG effect, at least in the case of these input.

In the inverted ordering case we find no viable points. After fitting the models' predictions on $\{\theta_{12}, \theta_{13}, \theta_{23}, \delta_{\text{CP}}, \Delta m_{21}^2, \Delta m_{32}^2\}$ to their global fit values, we get a $\chi^2_{\text{min}}/\text{NDF} \simeq 12/1$ in the inverted ordering case, indicating that the model is not a suitable description of the data in this case.

The model also gives predictions on the not-yet observed quantities. For example, the Dirac CP violating phase is predicted to be in the range $[256.72°, 283.33°]$, and the Majorana phases are: $\alpha_1 \in [128.03°, 233.58°]$, $\alpha_2 \in [0.30°, 130.21°] \cup [230.59°, 359.43°]$. The lightest neutrino mass is $m_1 \in [0.003, 0.010]$ eV. The effective Majorana neutrino mass in neutrinoless double beta decay is $|\langle m_{ee}\rangle| \in [0.00001, 0.010]$ eV.

To sum up, the S_4 model we built are elaborately controlled at a percent level of accuracy to render the mass matrix structure dictated by the SC mixing. Also as a result of the control, there are few free parameters left in the model. A numerical study of the parameter space shows that the model gives realistic predictions on neutrino masses and mixings, and can be tested in future experiments.

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