Primordial Magnetic Fields from a Cyclical Universe

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Abstract

The origin of cosmological magnetic fields is investigated in the context of a cyclical universe. Both the minimal and non-minimal coupling between electrodynamics and gravity are studied, applied to the phases of bounce and contraction of an eternal universe. We report magnetic field creation from a small seed electrical field both during bounce and contraction, being negligible in the first case but significant on the latter. We show how the contraction of the universe can be responsible for generating magnetic fields with strengths of the order of current bounds on extragalactic magnetic fields.

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1. INTRODUCTION

The study of the universe as a dynamical entity, comprising the observable reality that we can apprehend, began after physical cosmological models were built. This was possible after not only the discovery of the expansion of the universe [1, 2] but also the formulation of general relativity by Albert Einstein [3, 4], based on the deformation that mass-energy imparts to the space-time structure, which offers the most satisfactory description of gravity given the empirical evidences that sustain it. In order to explain observations, within Einstein's general relativity, the current standard cosmological model needs however to resort to the postulation of dark components for which evidence has yet to be found [5]. To solve the problems of the ΛCDM model, great efforts have been placed in modelling dark matter and dark energy, at the same time that research in modifications and extensions to general relativity has been conduced [6]. It is in the fact that general relativity does not easily admit quantization that lies the strongest hint of it being an incomplete theory. There are several frameworks that have been modifying it. At the same time, several cosmological models beyond the standard ΛCDM have been explored. In particular, we bring the reader's attention to models of eternal universes, i.e. that undergo expansion, contraction, bounce and expansion again, which have been proposed using different mechanisms [7, 8, 9, 10, 11, 12, 13]. On the practical side, the advantages of cyclical models lie in addressing not only the current postulation of an initial singularity but also problematic features introduced by the Big Bang scenario that require additional mechanism, e.g. inflation, to make it consistent.

It has been shown that modifying the Lagrangian by adding terms of higher order to the curvature is a possible way to account for quantum gravitational effects [14, 15]. This way of modifying gravity offers us the possibility to explore viable cyclical cosmological models [16] and this work deals with magnetogenesis in the framework of such a cosmological model.

Magnetic fields in cosmology and astrophysics play several important roles, for example, in both large and small scale structure formation [17, 18] and in the description of particle acceleration through the interstellar and intergalactic media [19]. Despite observational difficulties, they have been detected at several different scales, from planetary to galactic cluster magnetic fields [20, 21]. However, the question of how they were created remains unsatisfactorily answered in the ΛCDM model. In the accelerated expansion low curvature regime that our universe is currently undergoing [22], it has not been possible to model the mechanism for creation of cosmological magnetic fields without resorting to beyond standard model ingredients [23]. Phase transitions offer an environment where out-of-equilibrium conditions could induce turbulent motion in the plasma and generate magnetic fields. Despite that, recent particle experiments have shown that cosmological transitions rather than first or second-order were likely to be cross-overs. Thus, the standard model does not easily accommodate magnetogenesis during cosmological transitions. From the scenarios for magnetic field generation in the early universe, the epoch of inflation is the most favoured. However, in order for the generated fields to be able to account for the current magnetic field strengths, the conformal invariance of the electromagnetic action has to be broken during inflation. At the same time, the strong coupling problem and backreaction on the background expansion have to be accounted for [24].

Tentative lower bounds on extragalactic magnetic fields suggest that these fields are remnant from primordial fields originated cosmologically rather than astrophysically. These have been obtained from the non-observation of the expected electromagnetic cascade emission of distant blazars [25] and in general that yields a constraint for the strength of today's magnetic field of $B_0 \gtrsim 10^{-12}$ T. A usual way to extend the standard model in inflationary magnetogenesis is based on the introduction of additional couplings, such as to a scalar field [26, 27, 28, 29, 30]. Several studies analysed non-minimal coupling between gravity and electrodynamics as a possible origin for magnetic fields and this hypothesis has also been considered in the context of bouncing cosmologies [31, 32, 33]. Bouncing models consider a finite scale factor minimum instead of a Big Bang singularity as the beginning of an expanding universe.

Our aim is to study magnetic field generation before inflation, in a non-singular bouncing universe, both using minimal and non-minimal coupling. We first analyse what happens during the short time interval in which the universe bounces and later the case of magnetogenesis during the era of contraction.

We present in § 2 the basics of the cosmological model that define the background geometry; in § 3 we present the electrodynamic equations and derive them for the non-minimal case in curved space. In § 4 magnetic field generation during the bouncing and contracting phases of the universe is presented. Concluding remarks to these results are found in § 5.

2. CYCLICAL COSMOLOGICAL MODEL

This work does not rely on a particular modification of gravity to make the cosmological bounce possible. The central assumption of this model is that by the end of the expansion phase of the universe, all its structures must have been disrupted such that the universe is in an empty state before the beginning of contraction. The fate of gravitationally bounded systems in models of accelerated expanding universe dominated by a dark energy with an equation of state $w < -1$ is indeed to be destroyed. This ensures that from one cycle to the next the entropy and density of the universe do not increase. Any specific modification that allows for a bounce and prescribes a nearly-empty universe at the beginning of the contraction cycle will therefore be suitable for the magnetogenesis mechanism here proposed.

Assuming the universe to be isotropic, we use spatially flat Friedmann-Lemaître-Robertson-Walker

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[dx^2 + dy^2 + dz^2 \right] , \tag{1}$$

where $g_{\mu\nu}$ is the metric tensor, t is the time coordinate and $a(t)$ is the scale factor. The Ricci scalar takes the usual form

$$R = 6\dot{H} + 12H^2 ; \quad H \equiv \frac{\dot{a}(t)}{a(t)} , \tag{2}$$

where H is the Hubble parameter and dotted variables represent the derivative with respect to time.

The general geometric conditions that our model must meet to analyse cyclical magnetic field generation will be presented next.

2.1. Bouncing Conditions

Prior to the currently observed expansion, a cyclic universe would have bounced from a contracting into an expanding state. The scale factor decreases during contraction until it reaches a minimum a_{min} at a time that we define as $t(a_{\text{min}}) = 0$ and subsequently increases. We assume a symmetric bounce around the minimum, beginning at a time $t = -t_B$ and ending at $t = t_B$. This minimum replaces the initial Big Bang singularity. As a consequence, the Hubble parameter grows from negative to positive, $\dot{H}(t) > 0$. This implies that the Ricci scalar needs to have a positive maximum and, therefore, $\ddot{R}(t) < 0$. The curvature scalar admits an expansion in time during the bounce since its timescale is quite short in comparison to the timescales of cosmological processes. Keeping the even powers of the expansion, that yields

$$R(t) = R_{\text{max}} + R_2 t^2 + \mathcal{O}(t^3) , \tag{3}$$

with $R_{\text{max}} > 0$ and $R_2 < 0$.

2.2. Contraction Conditions

The phase of contraction begins after expansion halts and when there is a turnaround in the moment when the scale factor reaches a finite maximum $t(a_{\text{max}}) = t_{\text{max}}$. It then decreases until contraction ends when higher order corrections to the curvature become significant and induce a minimum of the scale factor, $t(a_{\text{min}}) = t_{\text{min}}$, causing the bounce.

It is important for a viable eternal non-singular cosmology that at the end of the expansion phase the universe can be effectively considered empty. Only if contraction begins after the universe's structures have been ripped apart and matter and energy densities diluted enough to be negligible will the bounce at the end of the contraction phase occur in the same conditions as in the previous cycle. This ensures that as cycles go by, entropy and densities do not grow exponentially but remain finite. This is crucial to avoid problems present in contracting matter dominated universes, such as a significant increase in entropy and the Belinsky-Khalatnikov-Lifshitz instability [37].

Through Friedmann's equation, we can compute the Hubble parameter

$$H^2 = \frac{\kappa}{3} (\rho + \rho_{\text{eff}}) , \tag{4}$$

where κ is Einstein's gravitational constant, ρ is the average energy density of the universe and ρ_{eff} describes the effect of higher curvature terms as an effective energy density. We can see here that no particular theoretical assumption is made about the model that could modify the Friedmann equations in order to enable the bounce and turnover into contraction. We simply consider that the effects of quantum gravity can be modelled by introducing a new effective contribution to the Friedmann equation, ρ_{eff}, which has an arbitrary time dependence. In fact, it is known that many proposed modifications, including various models of modified gravity, have an effect that can be expressed in this form. The term that includes ρ_{eff} then must be consistent with the previously discussed conditions that $H(t)$ must fulfil in order to obtain a turnaround. Specifically, we know that the Hubble parameter vanishes at the points $a_{\max}$ and $a_{\min}$, and that it should have a negative sign in between. In the regime when the universe begins to contract, these properties can be easily encapsulated with a simple test-function, obtained through an expansion in time of the form

$$\rho_{eff}(t) = b_0 + b_1 \frac{t - t_{\max}}{t_{\min} - t_{\max}} + b_2 \left(\frac{t - t_{\max}}{t_{\min} - t_{\max}} \right)^2 + \mathcal{O}(t^3) . \quad (5)$$

Setting, for the sake of symmetry, $b_0 = 0$ and through the condition that $b_1 = -b_2$ for $\rho_{\rm eff} > 0$, we obtain only one free adjustable parameter.

The approximation $\rho = p = 0$, with p the average pressure of the universe, follows from the empty state of the universe at the beginning of the contraction regime. Eq. (4), and consequently the scale factor, will have analytical solutions in this case. We have also checked the numerical solution in case the energy densities of matter and radiation evolve in time according to $\rho(t) = \rho_{mat}(t) + \rho_{rad}(t) = \rho^0_{mat}/a(t)^3 + \rho^0_{rad}/a(t)^4$, with ρ^0_{mat} and ρ^0_{rad} the values of the matter and radiation densities at the beginning of the contraction phase. For these initial ρ^0_{mat} and ρ^0_{rad} to be consistent with the assumption of a nearly empty universe at the beginning of the contraction phase, their values will be small enough such that throughout contraction the analytical approximation provides a very good description.

3. ELECTRODYNAMICS IN CURVED SPACE

A possible way to prescribe the electromagnetic field such that its evolution be easily traceable is to assume independent time and space evolution for the fields, e.g.

$$E_x = \phi(t)E(y,z)\,,\; E_y = \phi(t)E(x,z)\,,\; E_z = \phi(t)E(x,y)\,, \quad (1)$$

$$B_x = \psi(t)B(y,z)\,,\; B_y = \psi(t)B(x,z)\,,\; B_z = \psi(t)B(x,y)\,, \quad (2)$$

where $\phi(t)$ and $\psi(t)$ are functions of time, E_i and B_i the cartesian coordinates of the electric and magnetic field, respectively.

3.1. Non-minimal Electrodynamics

Using quantum electrodynamics, effects of vacuum polarization in curved space-time were shown to cause the minimal Einstein-Maxwell Lagrangian to be modified [34]. With this motivation in mind, several authors have studied magnetogenesis in the context of non-minimal coupling. In Ref. [35] it is applied to inflationary magnetogenesis and in Ref. [33] to bouncing magnetogenesis. At the cost of increasing complexity, several couplings that generate unexpected effects can be at play, for instance between the electromagnetic sector and the Ricci scalar, the Ricci tensor and the Riemann tensor, such as shown in Ref. [36]. We chose to modify the Lagrangian in the simplest way possible, adding a term that couples the Ricci scalar and the electromagnetic tensor $F_{\mu\nu}$ through a coupling constant ϵ

$$\mathcal{L}_{nm} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{\epsilon}{4} R F_{\mu\nu}F^{\mu\nu} . \quad (3)$$

Varying the action $S_{nm} = \int d^4x \sqrt{-g}\mathcal{L}_{nm}$, where g is the determinant of the metric tensor, with respect to the induction tensor [36]

$$H^{\mu\nu} \equiv F^{\mu\nu} + \frac{\epsilon}{2}(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho})RF_{\rho\sigma} \quad (4)$$

$$= (1 + \epsilon R)F^{\mu\nu}\,, \quad (5)$$

one obtains the electrodynamic equation

$$\nabla_\nu H^{\mu\nu} = 0\,. \quad (6)$$

Let us now compute the covariant derivative of the induction tensorr [36]

$$\nabla_\nu H^{\mu\nu} = \partial_\nu H^{\mu\nu} + \Gamma^\mu_{\nu\lambda}H^{\lambda\nu} + \Gamma^\nu_{\nu\lambda}H^{\mu\lambda} \quad (7)$$

$$= \epsilon(\partial_\nu R)F^{\mu\nu} + (1 + \epsilon R)\left(\partial_\nu F^{\mu\nu} + \Gamma^\mu_{\nu\lambda}F^{\lambda\nu} + \Gamma^\nu_{\nu\lambda}F^{\mu\lambda}\right)\,, \quad (8)$$

where $\Gamma^\lambda_{\mu\nu}$ is the Christoffel symbol.

Upon computation, Eq. (8) explicitly yields the following set of equations

$$-(1 + \epsilon R)\left(\partial_x E_x + \partial_y E_y + \partial_z E_z\right) = 0 \quad (9)$$

$$\epsilon \dot{R} E_x + (1 + \epsilon R)\left[\partial_t E_x - \partial_y B_z + \partial_z B_y + 3HE_x\right] = 0 \quad (10)$$

$$\epsilon \dot{R} E_y + (1 + \epsilon R)\left[\partial_t E_y - \partial_z B_x + \partial_x B_z + 3HE_y\right] = 0 \quad (11)$$

$$\epsilon \dot{R} E_z + (1 + \epsilon R)\left[\partial_t E_z - \partial_x B_y + \partial_y B_x + 3HE_z\right] = 0 \quad (12)$$

where $\dot{R} = 6(\ddot{H} + 4H\dot{H})$.

Eqs. (9)-(12) represent the modified Gauss' law for the electrical field and Ampère's law. The remaining two Maxwell equations are derived from the Hodge dual of the electromagnetic tensor ${}^*F^{\mu\nu} = \varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}/2$. Using the respective conservation law, $\nabla_\nu {}^*F^{\mu\nu} = 0$, we obtain the following set of equations

$$\begin{aligned} \partial_x B_x + \partial_y B_y + \partial_z B_z &= 0 && (13)\\ \partial_t B_x &= \partial_z E_y - \partial_y E_z - 3HB_x && (14)\\ \partial_t B_y &= \partial_x E_z - \partial_z E_x - 3HB_y && (15)\\ \partial_t B_z &= \partial_y E_x \partial_x E_y - 3HB_z\,. && (16) \end{aligned}$$

Compared to (9)-(12), note that Faraday's and Gauss' laws have now not been modified due to non-minimal coupling.

Inserting (1) and (2) in the Gauss' law for the electric and magnetic fields, we obtain the following differential equations, respectively

$$\dot{\phi}(t) + \left(\frac{\epsilon\dot{R}(t)}{1+\epsilon R(t)} + 3H(t)\right)\phi(t) = w\psi(t)\,, \quad (17)$$

$$\dot{\psi}(t) + 3H(t)\psi(t) = u\phi(t)\,, \quad (18)$$

with w and u constants of integration. This set of equations can be solved by using the conditions adequate to the contraction phase and the Hubble parameter thereof. Just as the dual electromagnetic field can be obtained from mapping the components of the field $E_i \to -B_i$, so can it be used to obtain the constants

$$u = \frac{\partial_m E(i,m) - \partial_l E(i,l)}{B(l,m)}\,,\; w = \frac{\partial_l B(i,l) - \partial_m E(i,m)}{B(l,m)}\,, \quad (19)$$

that relate to each other through $w = -u$.

3.2. *Minimal Electrodynamics*

It is simple to recover standard minimal electrodynamics by switching off the non-minimal coupling constant, i.e. setting $\epsilon = 0$, in the equations presented in the previous section. In this case the usual Langrangian

$$\mathcal{L}_{EM} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\,, \quad (20)$$

governs Maxwell-Einstein equations, obtained when $\epsilon = 0$ in (9)-(12). The system to solve in order to obtain the time evolution of fields is now simplified to

$$\dot{\phi}(t) + 3H(t)\phi(t) = w\psi(t)\,, \quad (21)$$

$$\dot{\psi}(t) + 3H(t)\psi(t) = u\phi(t)\,. \quad (22)$$

4. MAGNETIC FIELD GENERATION

Solving the time evolution of electrical and magnetic fields using (17), (18), (21) and (22) we can investigate in an approximated manner if magnetogenesis could have taken place in an environment different than the expansion phase of the universe. For our observational universe, that would correspond to a time preceding the early universe.

Our basic assumption is to begin from a vanishing magnetic field. In this case, independently of the setting, it is easily seen that a necessary condition for the generation of magnetic fields is a small electrical field. If the strength of this seed electrical field is small enough, it is plausible for it to have been generated by moving charged particles that created local non-vanishing electrical currents. These could have been present, on the one hand during the bounce due to the high energy densities at the end of contraction and on the other hand, before contraction, produced in the ripping process before contraction. Here we will arbitrarily take its strength to be $E = 3 \times 10^{-7}$ V/m. We have also chosen $u = 1$ without loss of generality.

Electric fields, even if present primordially, were fastly dissipated in the early universe due to the large electrical conductivity of the hot plasma. Since this does not affect magnetic fields, the remnants of cosmologically produced magnetic fields possibly left imprints that we can observationally probe nowadays. Although they depend on the coherence length of the field, for reference we look for solutions that fulfil the lower limits suggested for extragalactic magnetic fields of $B(t_0) \gtrsim 10^{-12}$ T at present [25]. Fields decay with expansion according to $B(t) \propto a(t)^{-2}$. We can then estimate the strength of primordial fields if expansion were the only decaying mechanism at play. Since from the Planck scale it is estimated that the scale factor increased 10^{32} times until today, the strength of magnetic fields at the Planck scale needed to have been around 10^{44} T. Being this the smallest scale that our current approaches can sustain, this estimate can be used as an approximation to the necessary field strength after the bounce.

Both during bounce and contraction, the term in Eqs. (17)-(18) that stems from non-minimal coupling remains negligibly small in all relevant solutions. For a non-minimal coupling as described by Eq. (3), this makes the non-minimal and minimal treatments equivalent to our purposes. We will therefore show the results in terms of the minimal approach and the reader should be reminded that the non-minimal solutions yield the same results.

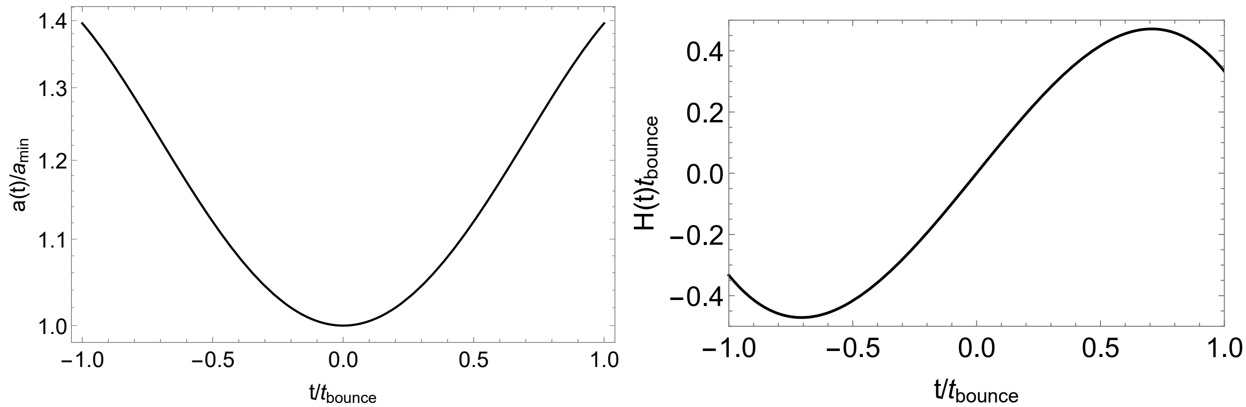

FIGURE 1: Scale factor and Hubble parameter during the bouncing phase of the universe.

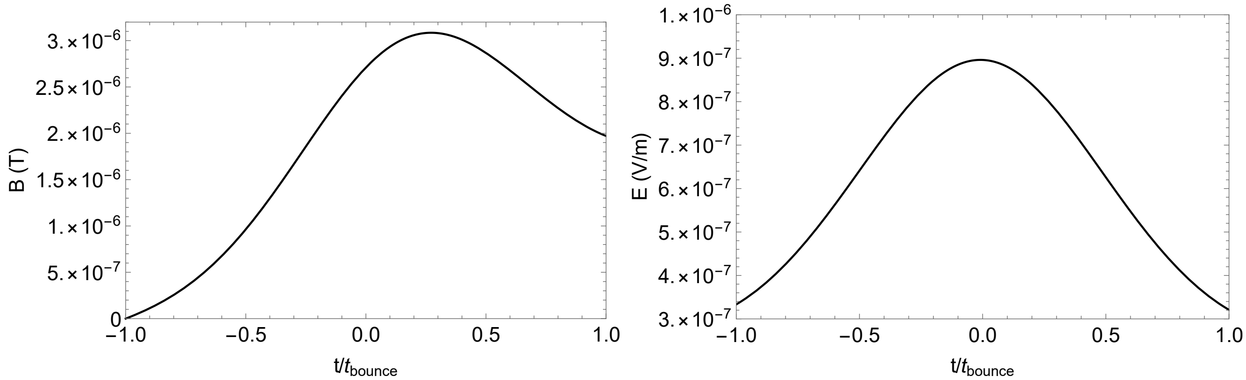

FIGURE 2: Magnetic and electrical field evolution during the bouncing phase of the universe.

4.1. Bouncing Phase

For simplicity (21)-(22) are cast in a dimensionless form by normalizing time to the bounce time t_{bounce}, which remains as a free parameter depending on the interval of time that the bounce takes, from $-t_{\text{bounce}}$ to t_{bounce}, with $t(a_{\text{min}}) = 0$. This in its turn depends on details of the particular bouncing model, making this analysis suitable for multiple cases. In Eq. (3), we fit the parameters R_{max} and R_2 to allow for maximal field growth, finding for the Hubble parameter and scale factor the curves shown in Figure 1. The obtained electromagnetic field is shown in Figure 2. In this setting, it can be concluded that from a small seed electrical field a magnetic field is induced, with maximal value at a_{min}. The geometrical symmetry of the bounce period allows for fields to grow as the scale factor diminishes, but similarly, to decay as the scale factor grows after the bounce takes place. The generated field will be rather small thanks to the short duration of the bounce in which the scale factor varies in a rather small amount. Thus, the geometrical properties that allow for magnetic field production are the same imposing a limitation on the primordial field strength. Therefore, the obtained values of magnetic field at the end of the bounce period are greatly inferior to what would be necessary to account for the lower bounds presently posed to extragalactic magnetic fields.

4.2. Contraction Phase

Eqs. (21)-(22) and respective parameters are now normalized using the time at which the scale factor is minimal, t_{min}. As can be seen in Figures 3 and 4, the decrease of the scale factor during contraction is accompanied by an electromagnetic field growth. The choice of parameters in Eq. (5), related to the magnitude of the Hubble parameter, has no influence in the shape of the curves $\phi(t)$ and $\psi(t)$, but it determines the strength of the generated magnetic field. We chose it such that it allowed for a field growth that reproduces the presently expected extragalactic magnetic fields. We thus conclude that during contraction primordial magnetic fields can be generated due to a seed electrical field. Unlike the case during the bounce, fields can suffer strong amplification in the favourable conditions of the contraction phase. That easily allows for the production of fields with the magnitude expected that primordial magnetic fields have in the early universe.

5. CONCLUSIONS

This work focused on studying the creation of magnetic fields in phases preceding the current cosmological cycle. Its main result is that primordial magnetic fields of strengths sufficient to account for the observational present bounds can be accomplished during the contracting phase of the universe.

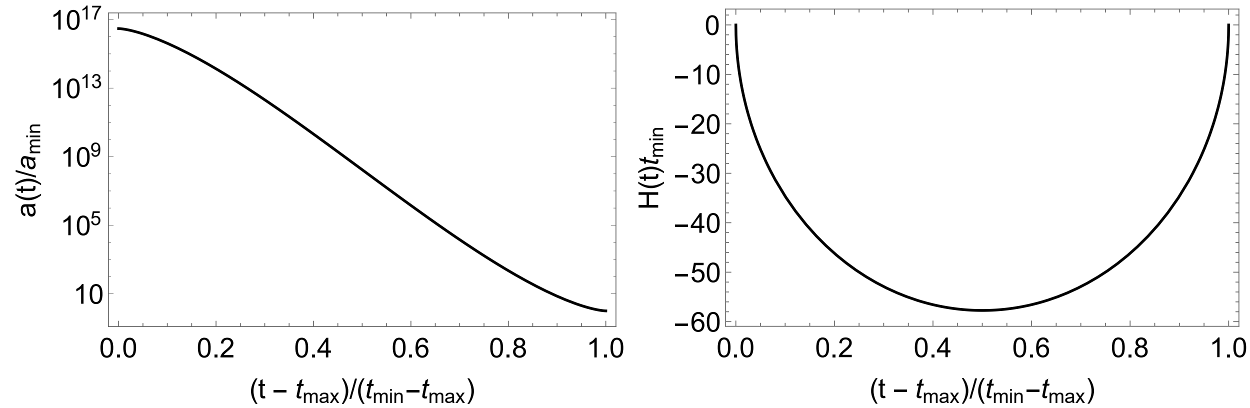

FIGURE 3: Scale factor and Hubble parameter during the contraction phase of the universe.

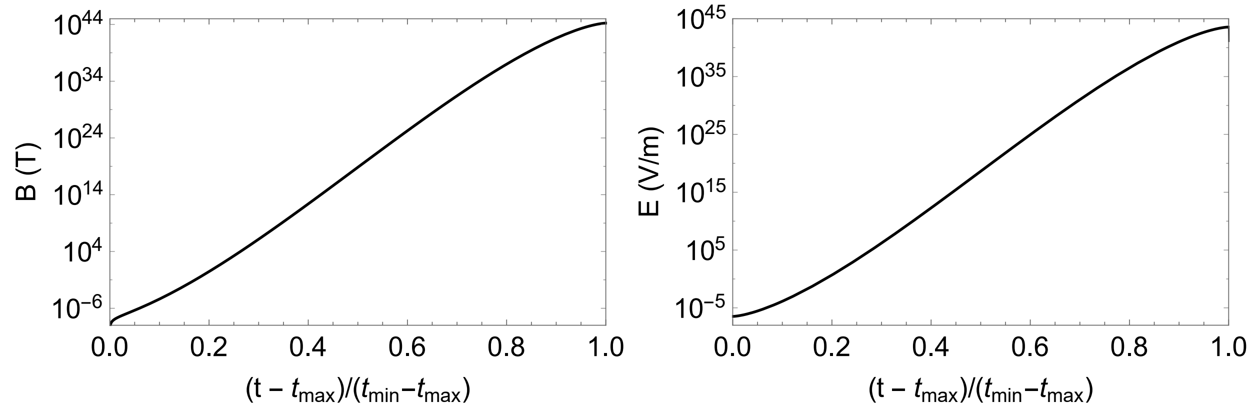

FIGURE 4: Magnetic and electrical field evolution during the contraction phase of the universe.

We opted to follow an eternal cyclical cosmological model where standard gravitation is modified by introducing higher order curvature corrections, such as described by Ref. [16]. In comparison to other magnetogenesis hypotheses in non-singular cosmologies, we do not resort to modifications of gravity that couple additional scalar or tensorial fields with the electromagnetic sector. In comparison to inflationary magnetogenesis, the simple Maxwell-Einstein Lagrangian that we use does not break conformal invariance. We do not have to solve the strong coupling problem and we are free from the backreaction of the field on the background expansion, questions that arise in the context of inflationary scenarious.

During the bounce of the universe, from an initially vanishing magnetic field and a seed electrical field, a magnetic field is generated. Its magnitude depends on the variation of the scale factor, which at the end of the bounce that corresponds to the beginning of the expansion phase, is small in comparison to the fields expected to be present in the early universe in order to derive the present-day bounds.

On the other hand, during contraction, as presented in § 4.2 we report the creation of magnetic fields of strengths of $B(a_{\min}) \approx 10^{44}$ T. This field strength would decay during expansion until resulting in the present remnant cosmological magnetic fields. This result can be easily generalized to yield different field strengths, according to the freedom in Eq. (5) of the parametrization of the Hubble parameter evolution during contraction. The solutions are fairly stable with respect to the initial conditions. According to our findings on the bounce region, an existing magnetic field at the end of the contraction phase is therefore likely to be mildly amplified during the bounce. This serves also to guarantee that the magnetic field generated during contraction is not washed out when curvature corrections become significant and induce the bounce. Thus it can survive to the expansionary phase. We have also found that a simple coupling between curvature and the field tensor did not influence the results in comparison to the minimal coupling.

The central assumption of our model consists in considering that at the beginning of a new cosmological cycle, when the universe begins to contract, it is in an empty state. An accelerated expansion of the type that we are currently undergoing might lead to such scenario. It also enables vacuum electrodynamics to be a good framework for our treatment.

The proposed mechanism counts on the presence of a non-vanishing seed electrical current that could be locally created during the contraction phase of the universe, which would be sufficient to trigger the creation of a magnetic field. This small seed magnetic field would quite simply suffer amplification following the geometrical properties of contraction. As the scale factor decreases, the generated magnetic field gets stronger. The field strength that can result at the beginning of the expansion phase depends on particular details of the description of the phase of contraction. This model is therefore flexible enough to encompass several beyond standard model possibilities in the light of cyclical cosmologies. Despite its generality, it offers predictive power regarding the long unsettled topic of magnetic field generation that we expect to soon be further probed by upcoming observations.

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